Image Processing

image formation, segmentation

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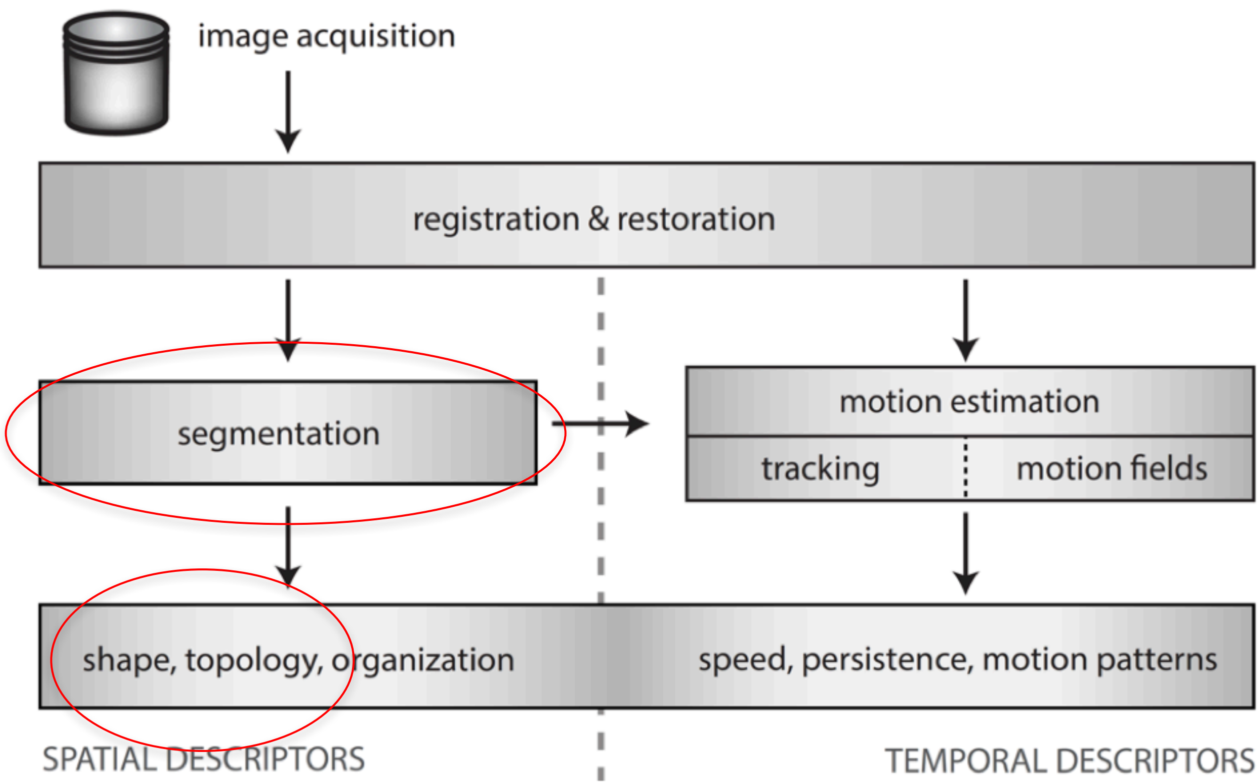
Outline



Computational methods for analysis of dynamic events in cell migration

Castañeda V, Cerda M, 2014

Outline

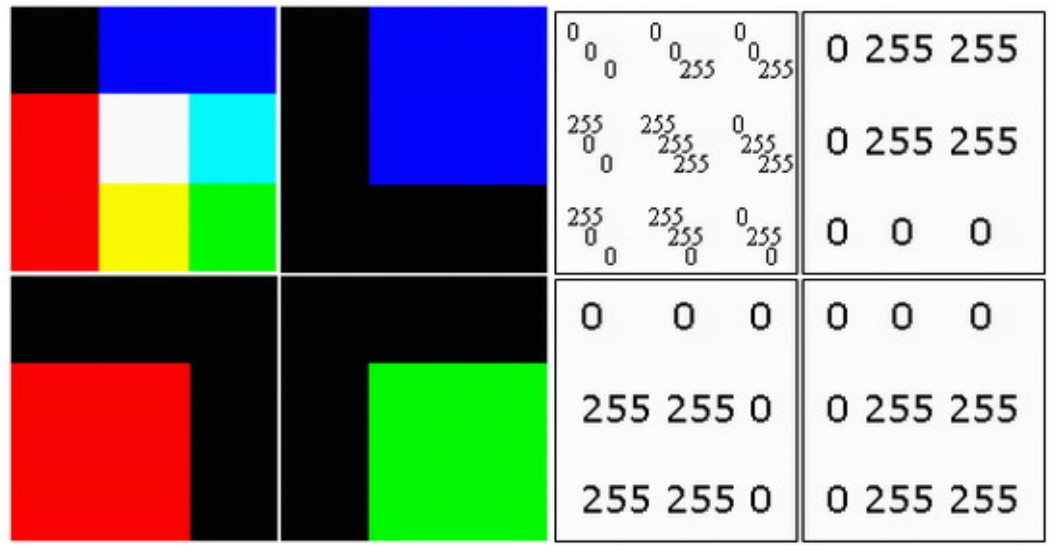
1. Introduction

    - Images
    - Segmentation*

2. Descriptors

    - Shape & Topology*
    - *Tracking*

- A digital image can be defined as a function over a discrete space
  - A typical 2D image model is the raster image: array (matrix) of pixeles in cartesian coordinates (*x*, *y*)
  - A numeric value for brightness (intensity) or color is associated to each pixel



$$I = f(x, y)$$

$$(x, y) \in [0, \dim_x - 1] \times [0, \dim_y - 1]$$

$$I\,[x_i, y_j] = f\,[x_i, y_j]$$

Digital Images

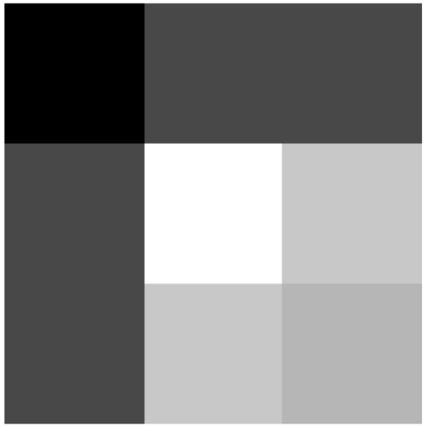
- ...so, a digital image can be treated as a function (in the mathematical sense)...
  - on a discrete domain
  - with numeric values associated to each elements, representing a property (such as color, brightness, depth, etc.)

Aubert & Kornprobst (2006)
Mathematical Problems In Image Processing

Digital Images

- Greyscale image
  - A brightness (intensity) level is defined for each pixel

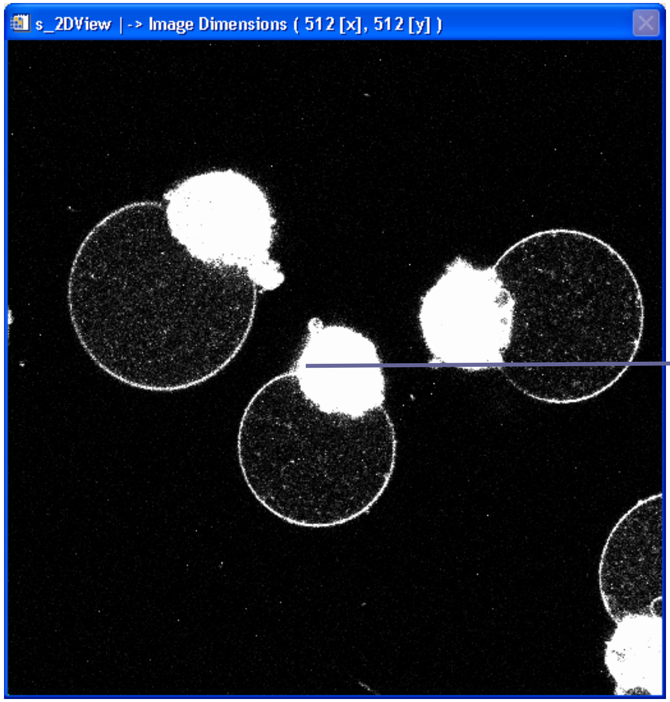
$$\begin{matrix} 0 & 85 & 85 \\ 85 & 255 & 170 \\ 85 & 170 & 85 \end{matrix}$$



$$I[x,y]$$

Digital Images

Binary value	Decimal value
0000 0000	0 (black)
0000 0001	1
0000 0010	2
0000 0011	3
0000 0100	4
0000 0101	5
0000 0110	6
0000 0111	7
0000 1000	8
…	…
1111 1011	251
1111 1100	252
1111 1101	253
1111 1110	254
1111 1111	255 (blanco)



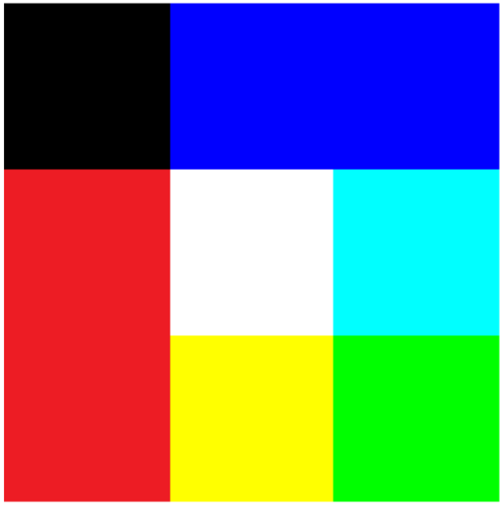
8 bit greyscale image

I(290,267) = 220

A *n* bit greyscale image encodes up to 2^n intensity values

Digital Images

- RGB image
  - Three channels for respective primary colors: Red, Green, Blue



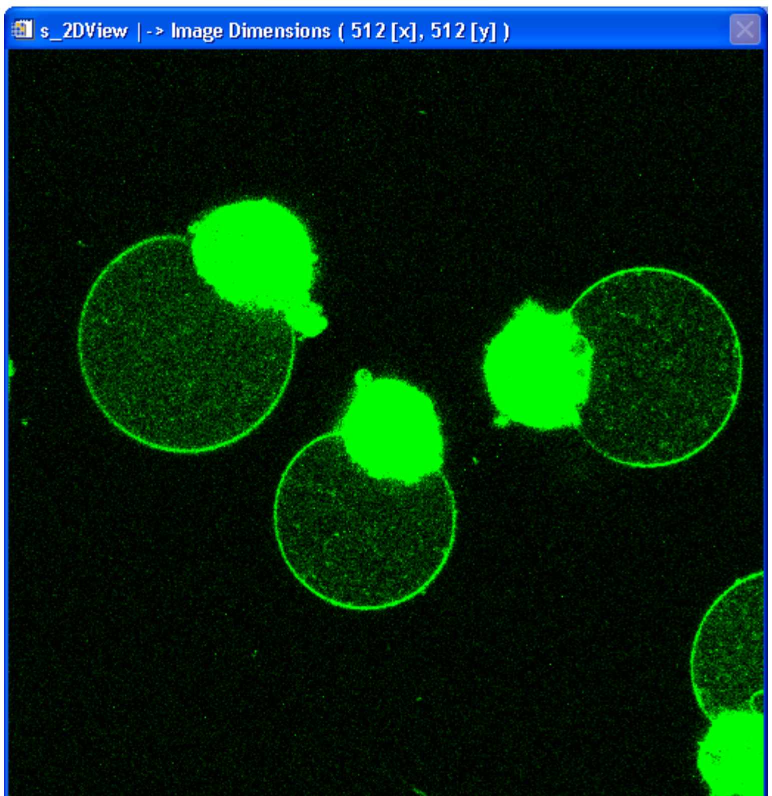
0	0	0
0	0	0
0	255	255
255	255	0
0	255	255
0	255	255
255	255	0
0	255	255
0	0	0

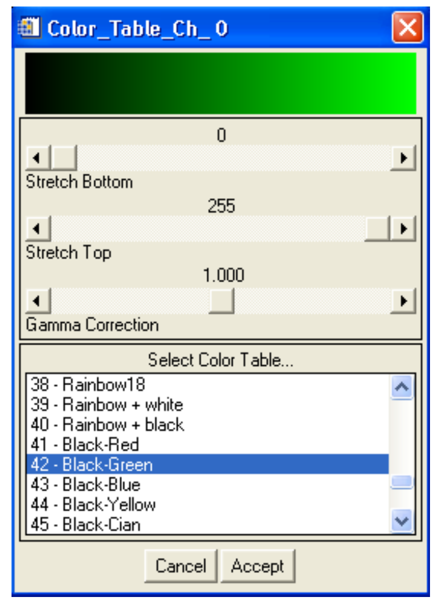
$$r[x,y]\; g[x,y]\; b[x,y]$$

- Other color spaces are HSV, LAB

Digital Images

- It is possible to define color tables (or lookup tables, LUTs) for visualization purposes. A grayscale image can be displayed using a green scale.



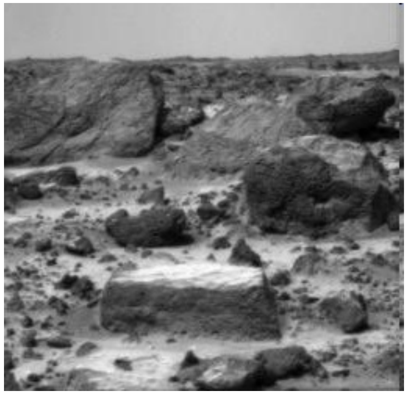


r	g	b
0	0	0
0	1	0
0	2	0
:	:	:
:	:	:
:	:	:
:	:	:
0	200	0
:	:	:
:	:	:
0	255	0

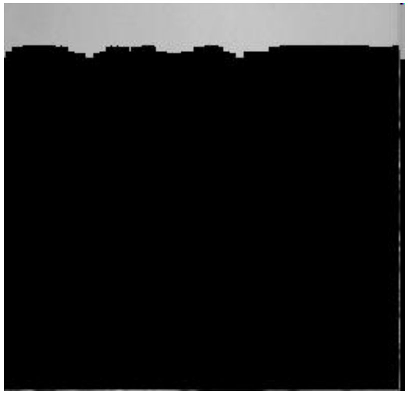
- Segmentation
  - The partitioning of a given image into regions of interest (ROIs) according to given criteria (e.g. color).
  - After segmentation, further characterizations can be performed upon the resulting ROIs.

Shapiro LG and Stockman GC (2001):
“Computer Vision”, pp 279-325
New Jersey, Prentice-Hall

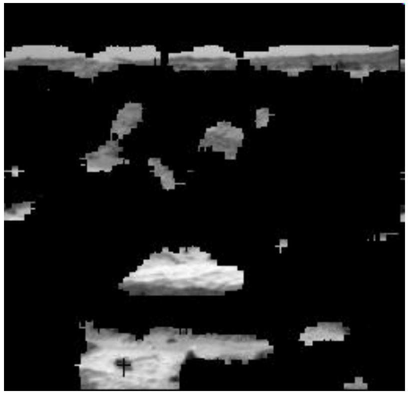
Segmentation



Sol 3, Mars
Pathfinder Mission

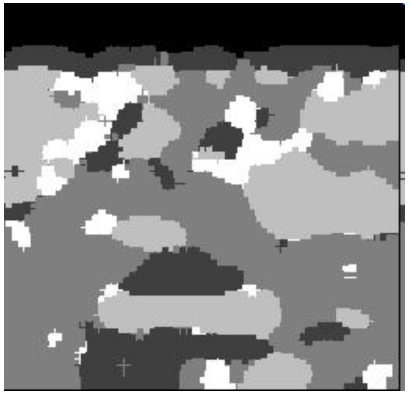


Sky / Flat



Dust / Horizon

...etc...

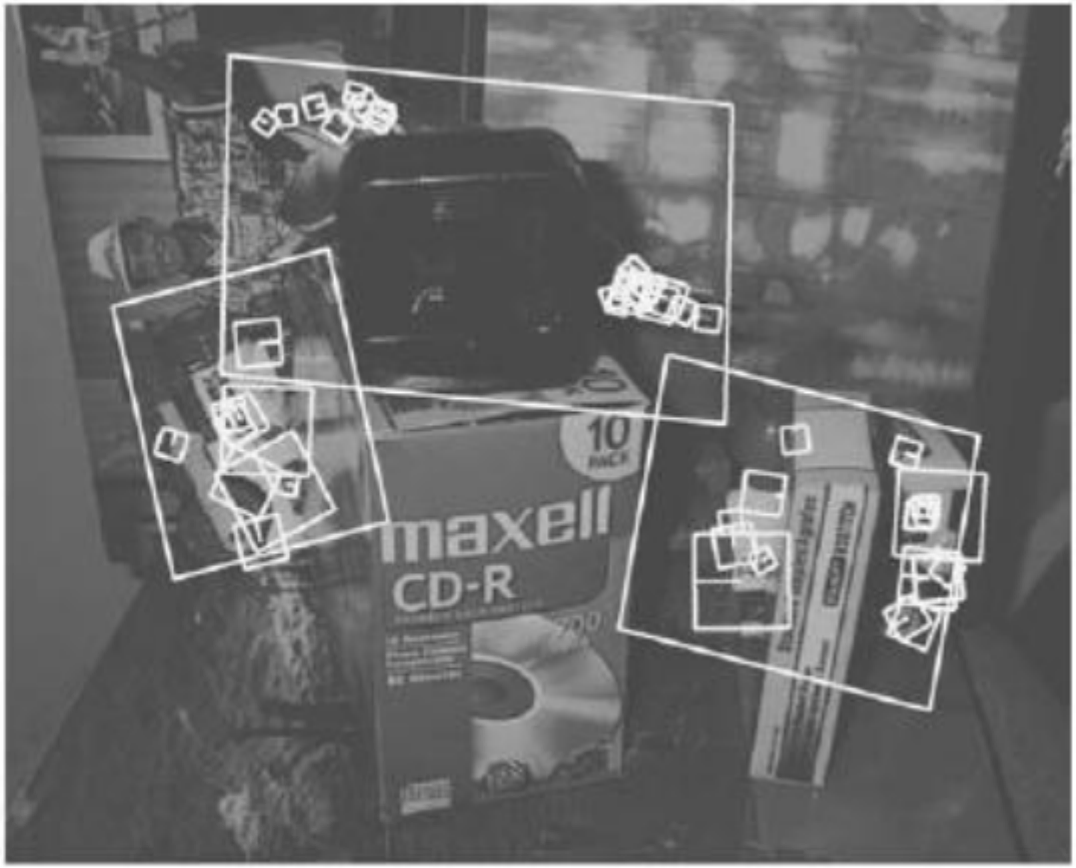


Final segmentation

Images from NASA

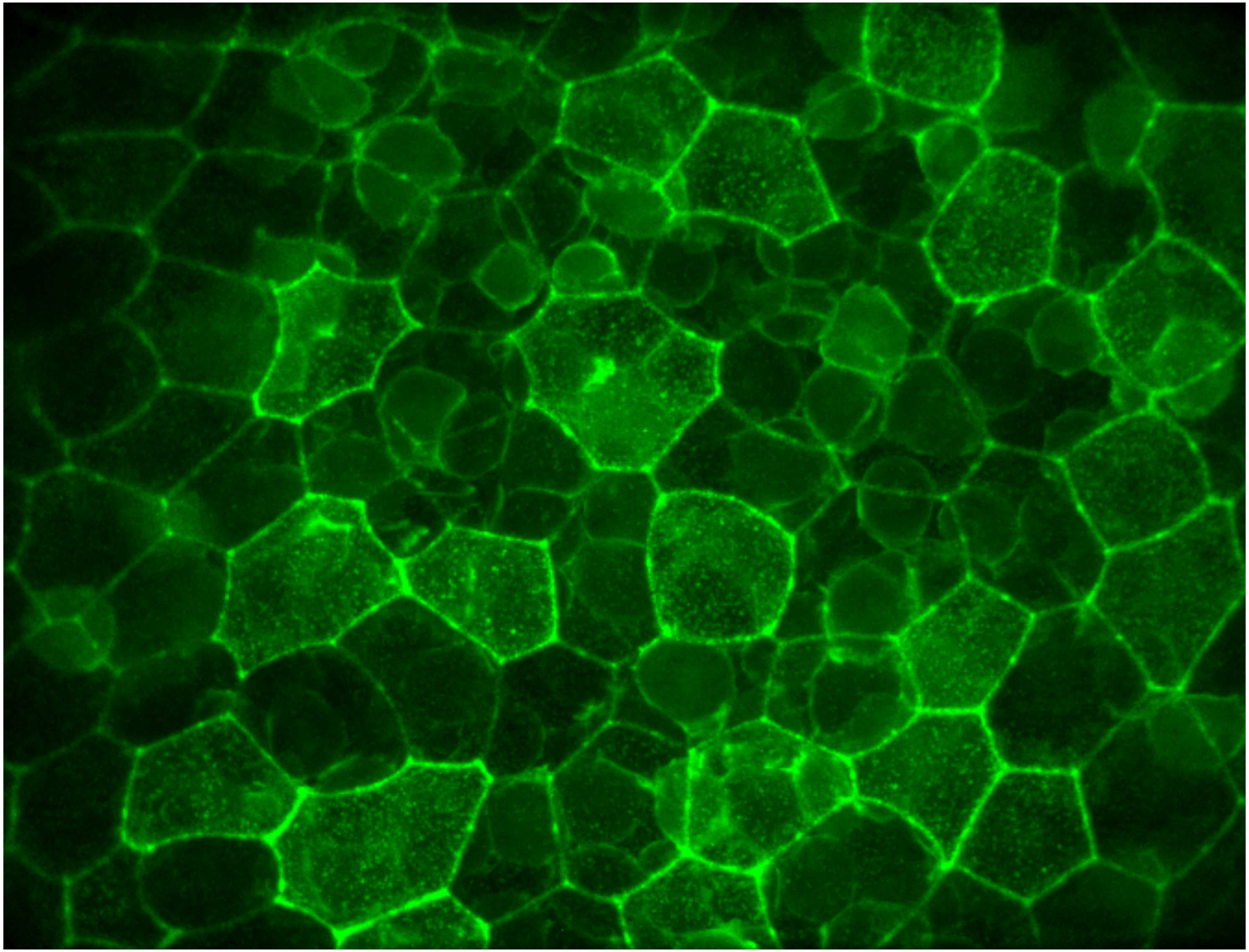
Segmentation

- Not only objects as ROIs, but features…



Scale Invariant Feature Transformation (SIFT), D Lowe (2004). Image from J Clemons (2009)

Segmentation



Max Z-Project from confocal microscopy of Fundulus N. [data by German Reig 2015]

Problems

- Lack of absolute criteria or standards (Ground Truth, Gold Standard [1,2])
- Missing or erroneus information (e.g. non-specific markers in samples)
- What to do? A “good” (i.e. carefully performed and controlled) acquisition ease this process

[1] Jason D. Hipp et all. Tryggo: Old norse for truth: The real truth about ground truth. New insights into the challenges of generating ground truth maps for WSI CAD algorithm evaluation. Pathol. Inform 2012, 3:8

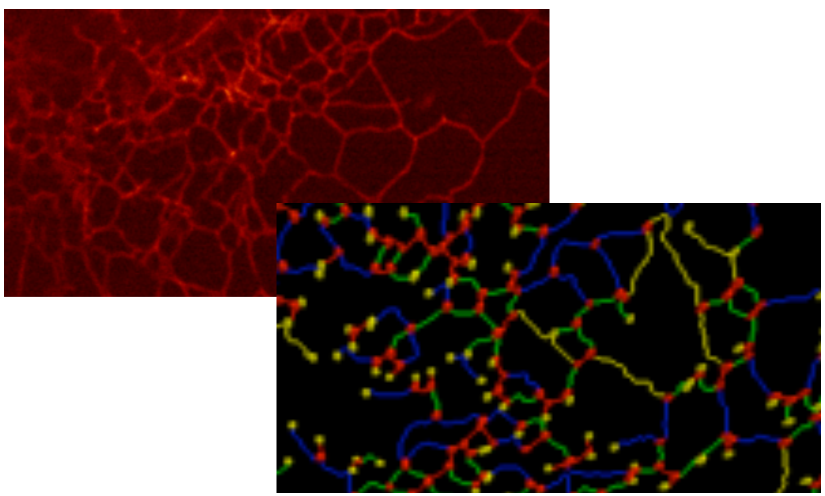
[2] Luc Bidaut, Pierre Jannin. Biomedical multimodality imaging for clinical and research applications: principles, techniques and validation. In Molecular Imaging:Computer Reconstruction and Practice (NATO Science for Peace and Security Series B: Physics and Biophysics), Springer, 2008, ISBN-13: 978-1402087516.

Segmentation

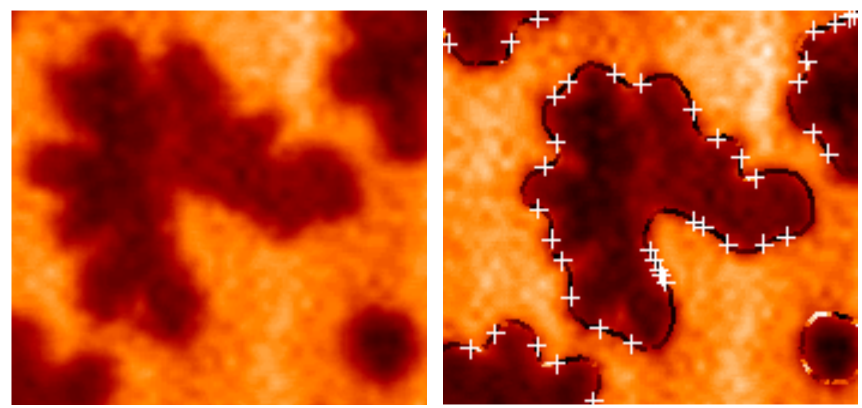
- Segmentation is the first step toward further quantifications
  - In addition to images, ROI models and data structures can suit for different types of descriptions

Parameter estimation…

- Size: area, perimeter
- Boundary: inflections, shape
- Topology: connectivity, endpoints



Endoplasmic reticulum in a COS-7 cell
O Ramírez, L Alcayaga (2012)



Lipid monolayers
J Jara (2006), Fanani et al (2010)

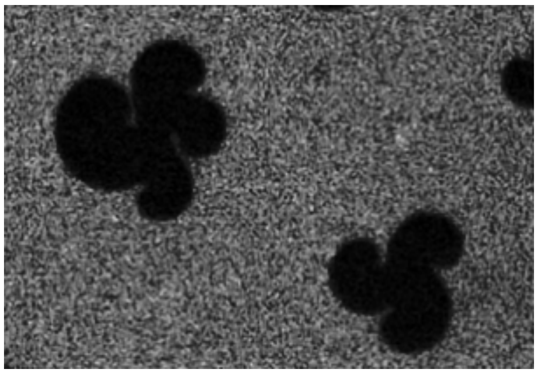
Segmentation

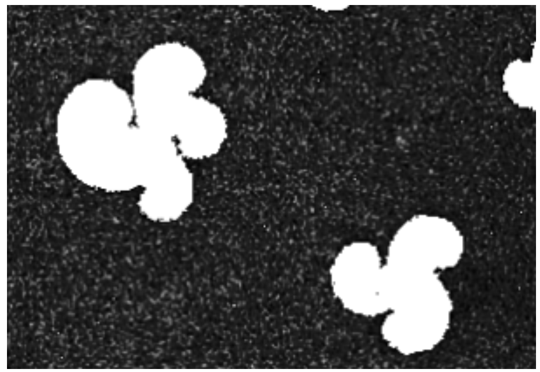
1. Classic approaches (filters)
   - Thresholding
   - Matrix convolution filters + threshold
   - Mathematical morphology

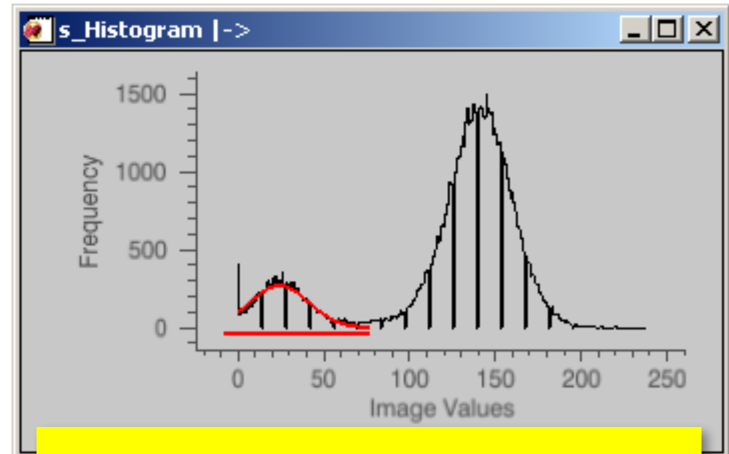
2. Advanced approaches
   - Shape priors (*pattern matching*)
   - Deformable models (active contours)
     - parametric
     - implicit

Segmentation – basics

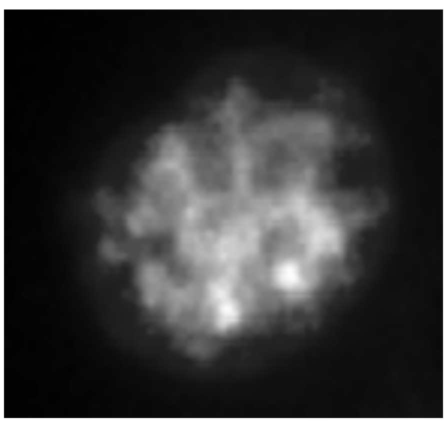
- Threshold filter
  segmentation: ROIs (white) / background (black)

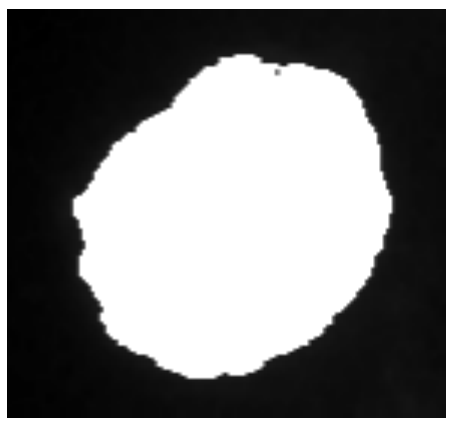


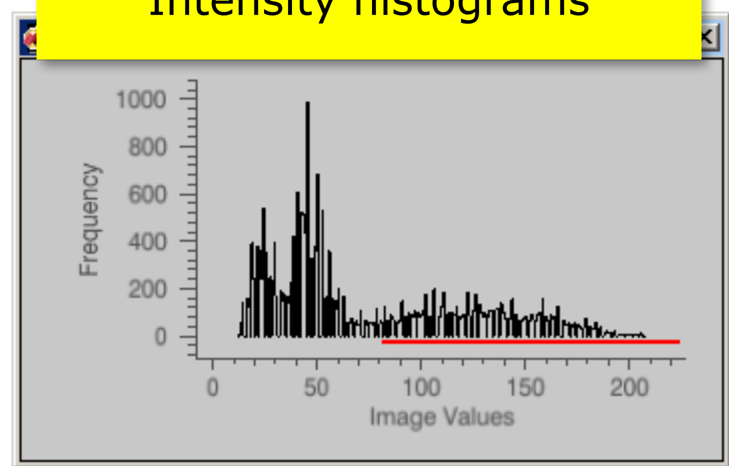




Intensity histograms







Segmentation – basics

- Otsu threshold
  - Idea: to separate the image pixel in two classes (sets), minimizing the sum of variances from both classes

$$\min \sigma_w^2(t) = \omega_1(t)\sigma_1^2(t) + \omega_2(t)\sigma_2^2(t)$$

$$t : \text{threshold}, \quad \omega_i : \text{probability of class } i$$

$$\omega_1 = \sum_0^t p(i) \qquad \omega_2 = \sum_{t+1}^{top} p(i)$$

Algorithm

1. Compute histogram and probabilities of each intensity level
2. Set up initial $\omega_i(0)$ and $\mu_i(0)$
3. Step through all possible thresholds $t = 1 \ldots$ maximum intensity
   1. Update ω_i and μ_i
   2. Compute $\sigma_b^2(t)$
4. Desired threshold corresponds to the maximum $\sigma_b^2(t)$
5. You can compute two maximums (and two corresponding thresholds). $\sigma_{b1}^2(t)$ is the greater max and $\sigma_{b2}^2(t)$ is the greater or equal maximum
6. Desired threshold = $\dfrac{threshold_1 + threshold_2}{2}$

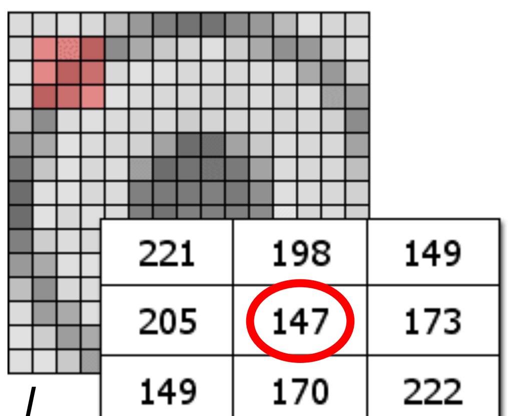
N. Otsu (1979).

Segmentation – basics

- Convolution
  - Lots of filters based on this principle
    http://en.wikipedia.org/wiki/Convolution
- Matrix convolution, in our case, is an operation between two matrices, namely…
  - the input image, *I*
  - a *kernel*, *K*

-1	0	1
-2	0	2
-1	0	1

K



I

$$(K \otimes I)_{i,j} = (-1 \cdot 221) + (0 \cdot 198) + (1 \cdot 149) + (0 \cdot 205) + (0 \cdot \mathbf{147}) + (0 \cdot 173) + (-1 \cdot 149) + (0 \cdot 170) + (1 \cdot 222) = -63$$

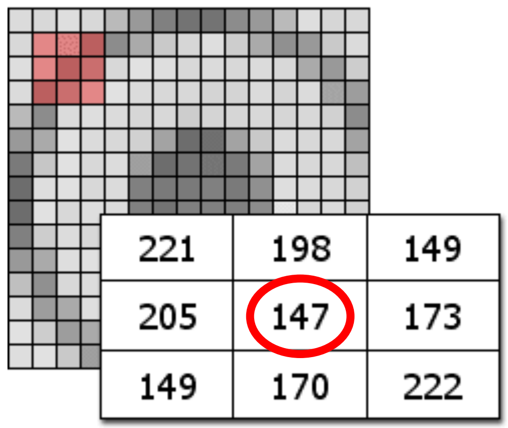
Adapted from James Matthews, 2002
http://www.generation5.org/content/2002/convolution.asp

Segmentation – basics

K

-1	0	1
-2	0	2
-1	0	1

I



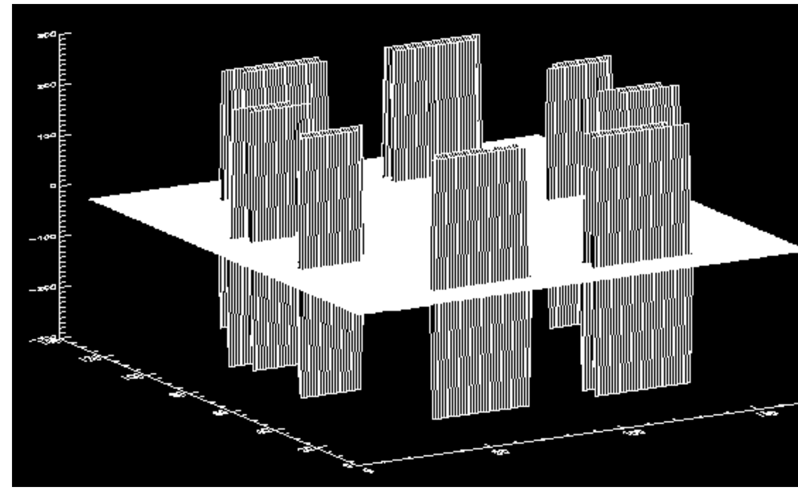
$$
\begin{aligned}
(K \otimes I)_{i,j} = {} & (-1 * 222) \\
& + (0 * 170) \\
& + (1 * 149) \\
& + (-2 * 173) \\
& + (0 * \mathbf{147}) \\
& + (2 * 205) \\
& + (-1 * 149) \\
& + (0 * 198) \\
& + (1 * 221) = +63
\end{aligned}
$$

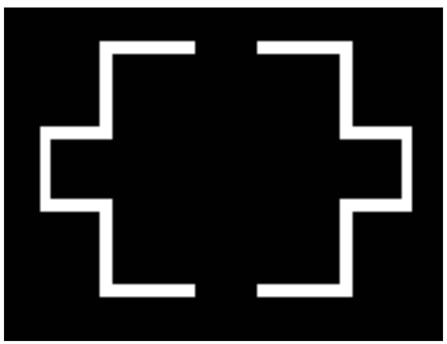
Matrix convolution can be implemented in different ways… beware of the algorithm!

James Matthews, 2002
http://www.generation5.org/content/2002/convolution.asp

- Intensity gradients (discrete approximation)

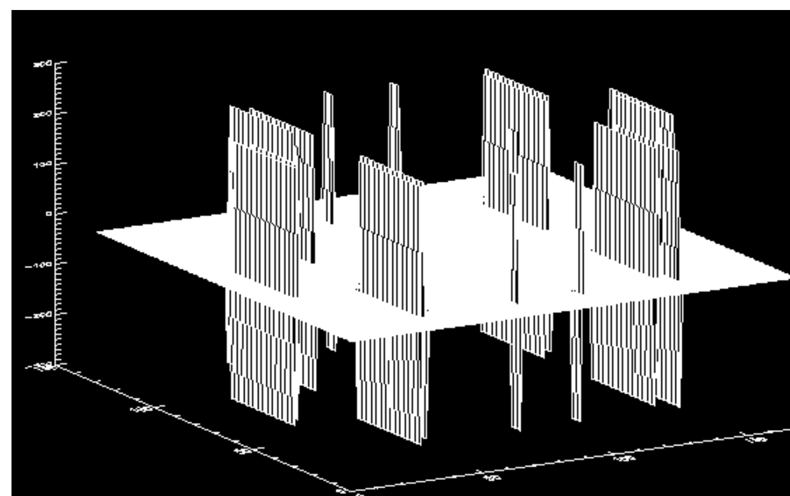
$$\frac{\partial I}{\partial x} \approx$$

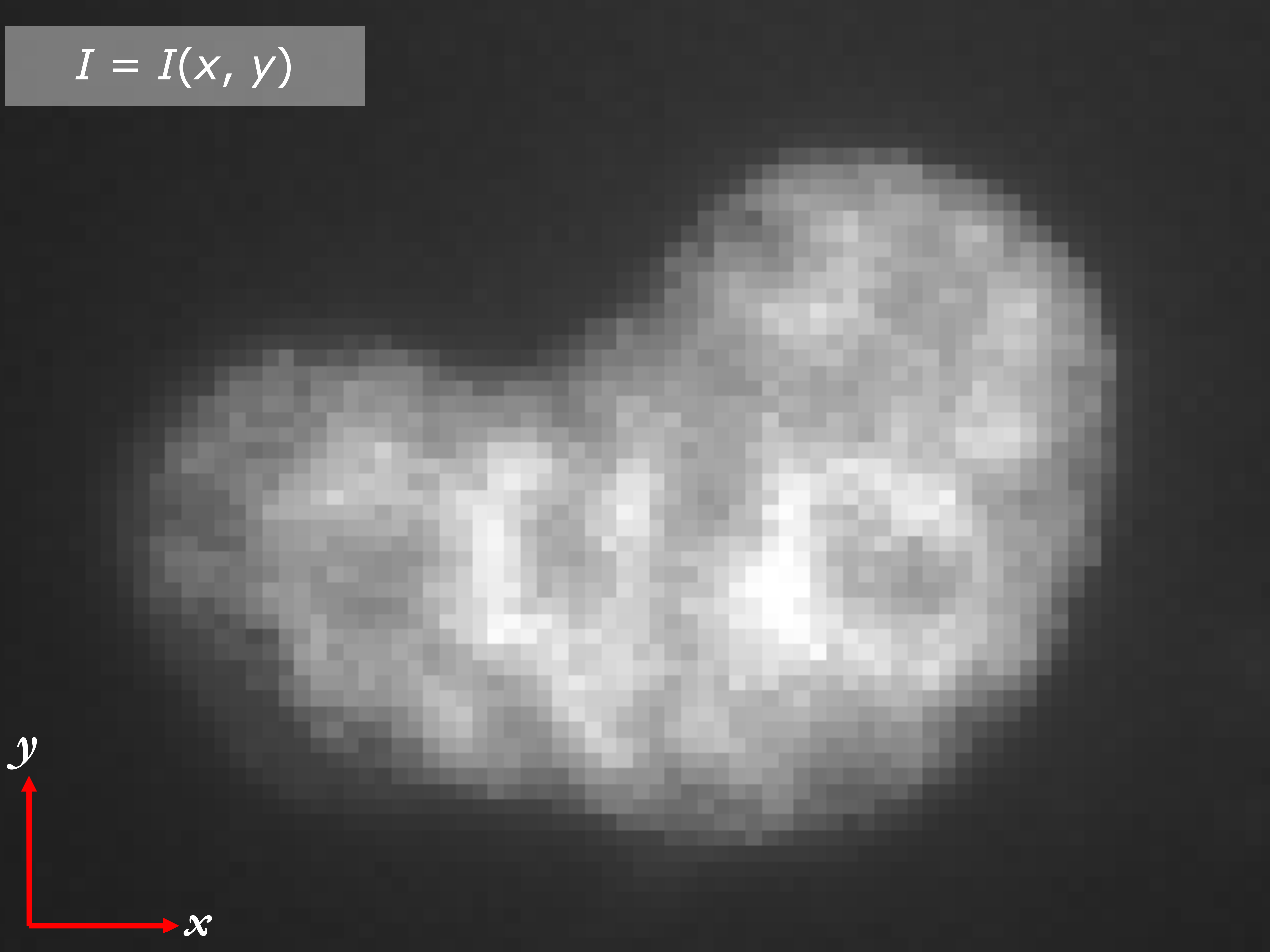




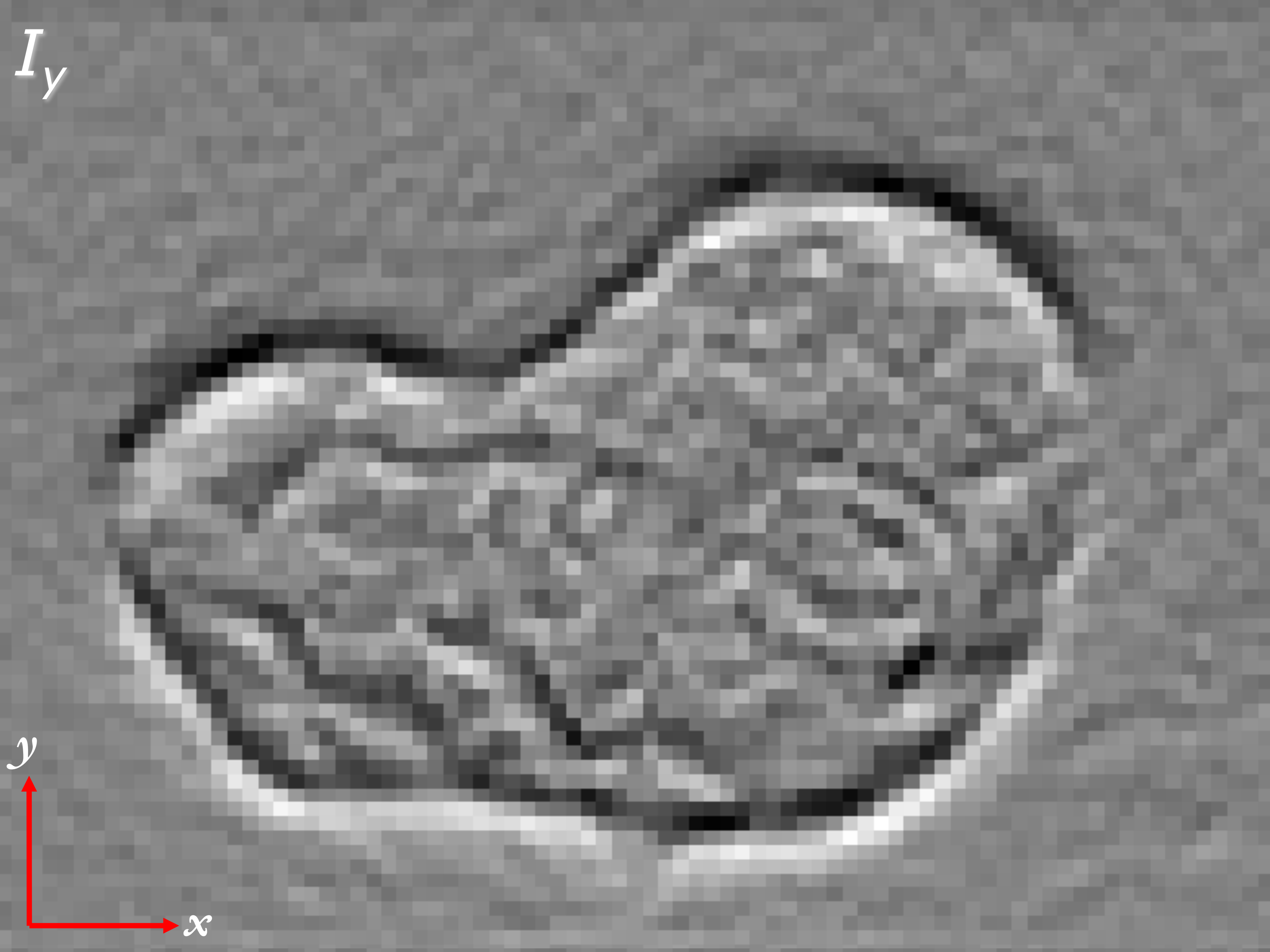
flat | + | -

$$\frac{\partial I}{\partial y} \approx$$

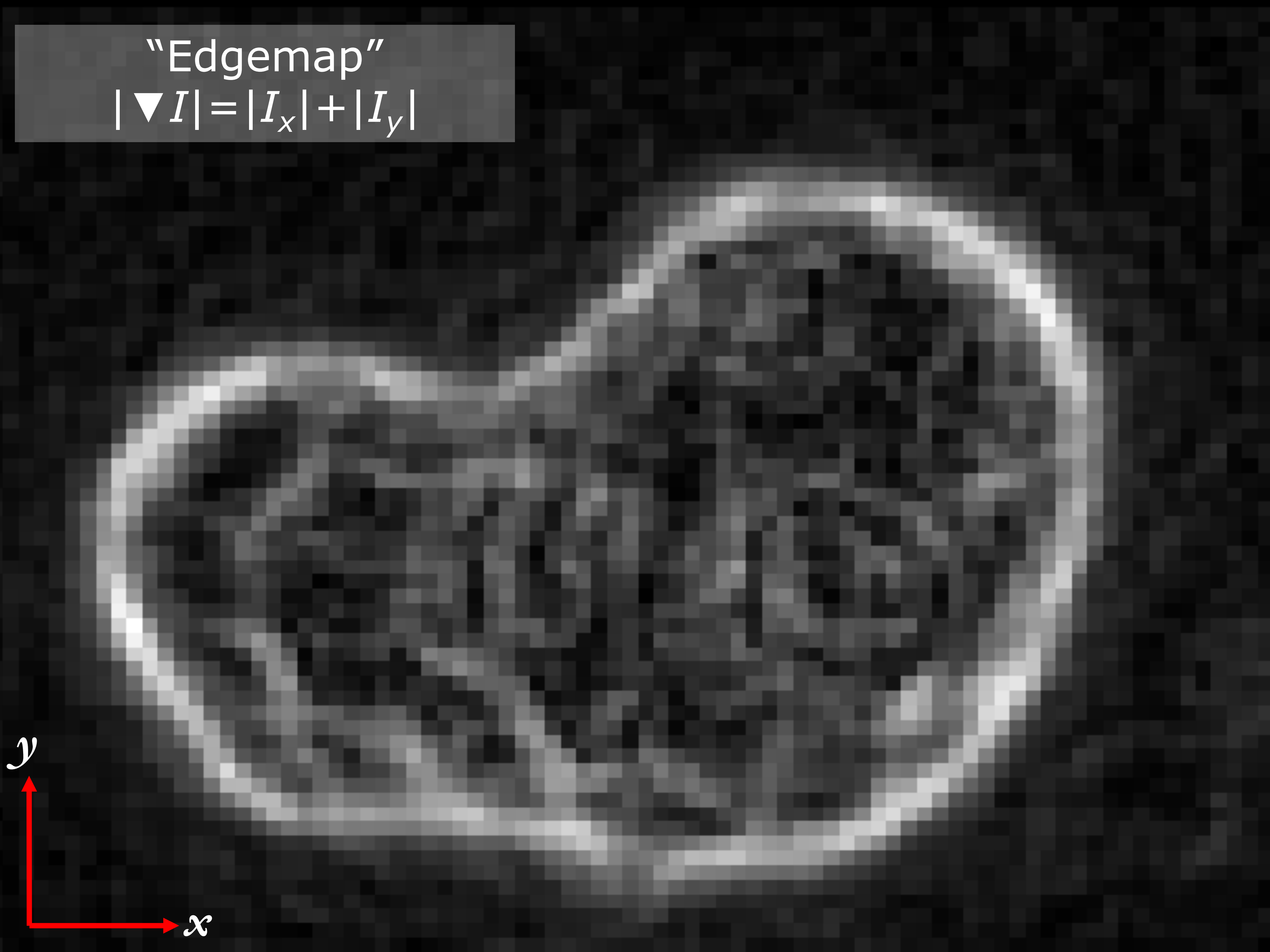




$I = I(x, y)$
y
x

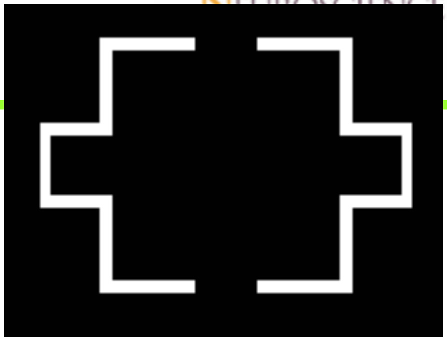


I_y
y
x


"Edgemap"
$|\nabla I| = |I_x| + |I_y|$
y
x

Segmentation – basics

- Intensity gradients (discrete approximation)

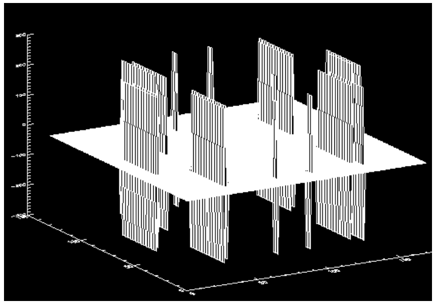


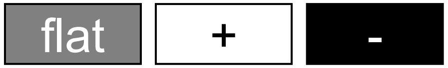
$$I = I(x, y)$$

$$\frac{\partial I}{\partial x} \approx \frac{I(x+\Delta x, y) - I(x, y)}{\Delta x} = Kx \otimes I$$

$\Delta x = 1\,\text{pixel}$

$$Kx = \begin{Bmatrix} 0 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{Bmatrix}$$

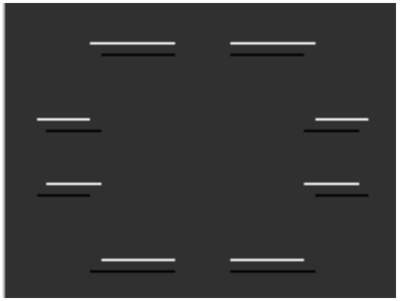


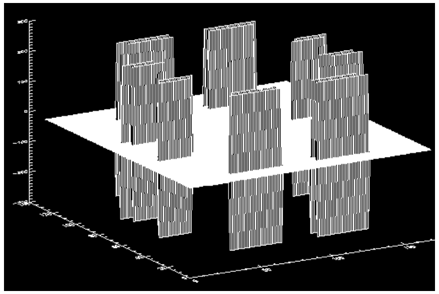


$$\frac{\partial I}{\partial y} \approx \frac{I(x, y+\Delta y) - I(x, y)}{\Delta y} = Ky \otimes I$$

$\Delta y = 1\,\text{pixel}$

$$Ky = \begin{Bmatrix} 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{Bmatrix}$$





Segmentation – basics

- Kernels…

$$I = I(x, y)$$

Laplace

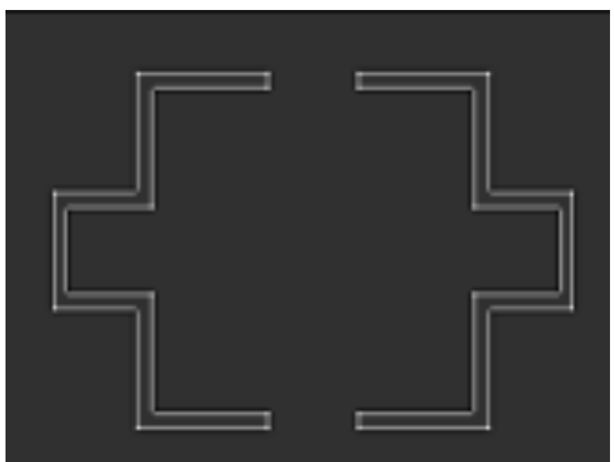
$$\nabla^2 I = \frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2}$$

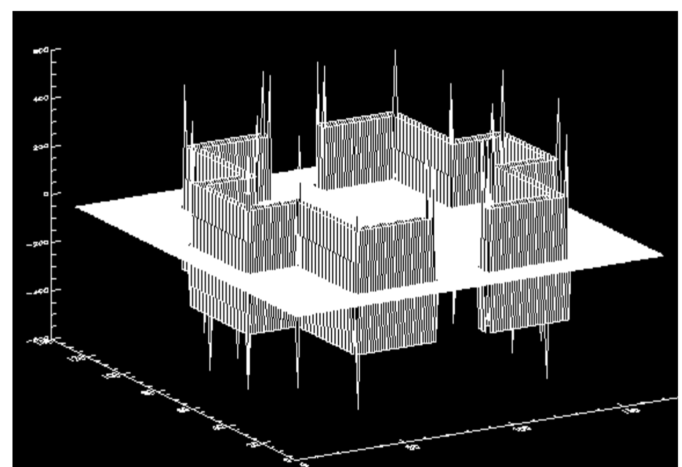
$$\nabla^2 I \approx \frac{f(x+\Delta x, y) - 2f(x,y) + f(x-\Delta x, y)}{(\Delta x)^2} + \frac{f(x, y+\Delta y) - 2f(x,y) + f(x, y-\Delta y)}{(\Delta y)^2}$$

$$\nabla^2 I \approx \frac{f(x+\Delta x, y) + f(x, y+\Delta y) - 4f(x,y) + f(x-\Delta x, y) + f(x, y-\Delta y)}{(\Delta x)^2} = K_L \otimes I$$

$$\Delta x = \Delta y = 1 \text{ pixel}$$

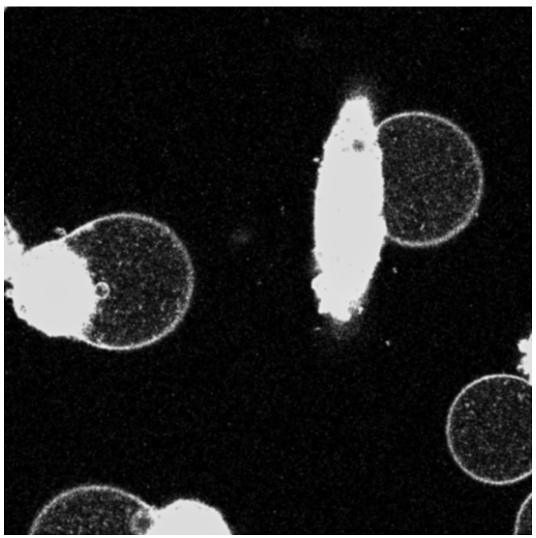
$$K_L = \begin{Bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{Bmatrix}$$



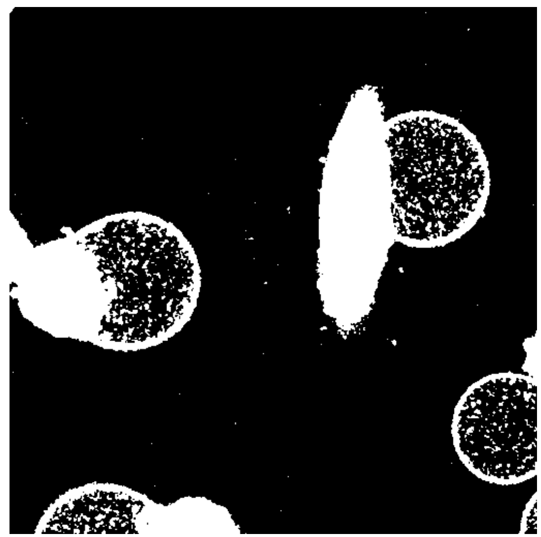


Segmentation – basics

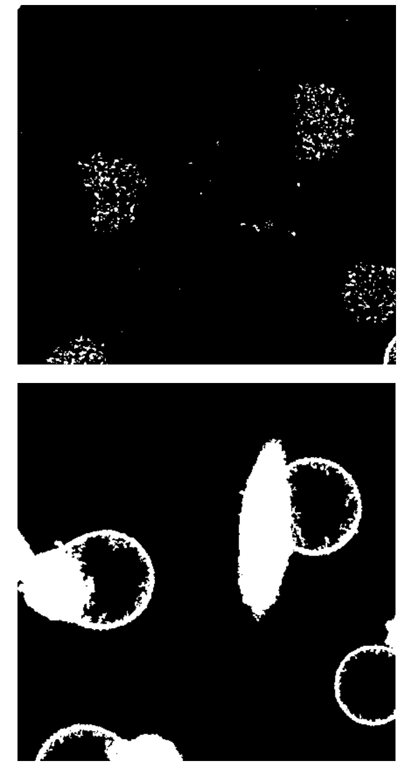
- Morphology based filters
  - Example: size selection



Input greyscale image



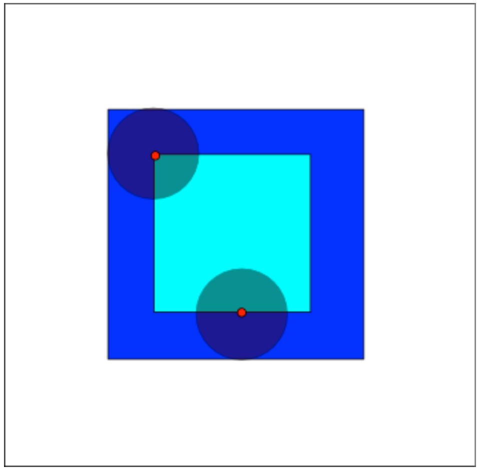
After thresholding…



Size selection

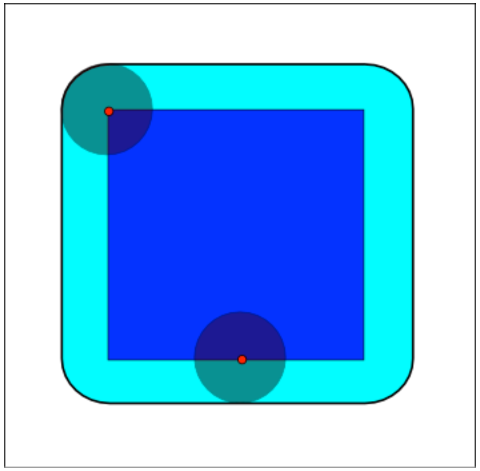
How to define a size-select algorithm?

- Mathematical morphology



Erode

$$A \ominus B = \{z \in \mathbb{R}^2 | B_z \subseteq A \}$$



Dilate

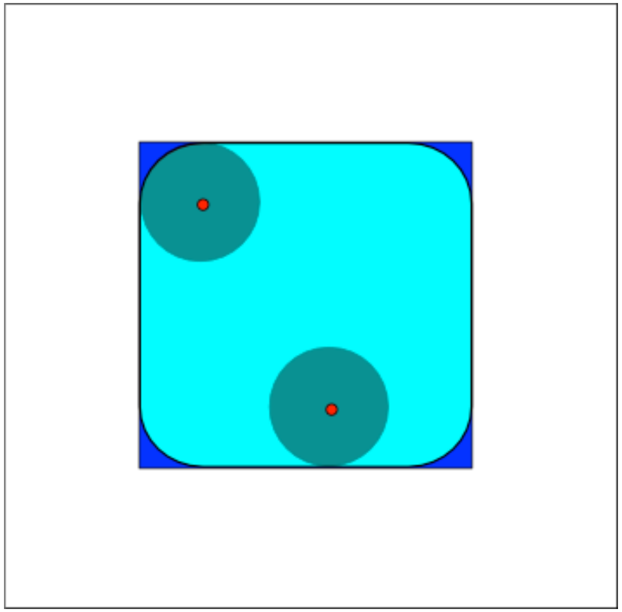
$$A \oplus B = \bigcup_{b \in B} A_b$$

What is B?

- Mathematical morphology

To open:

$$A \circ B = (A \ominus B) \oplus B$$

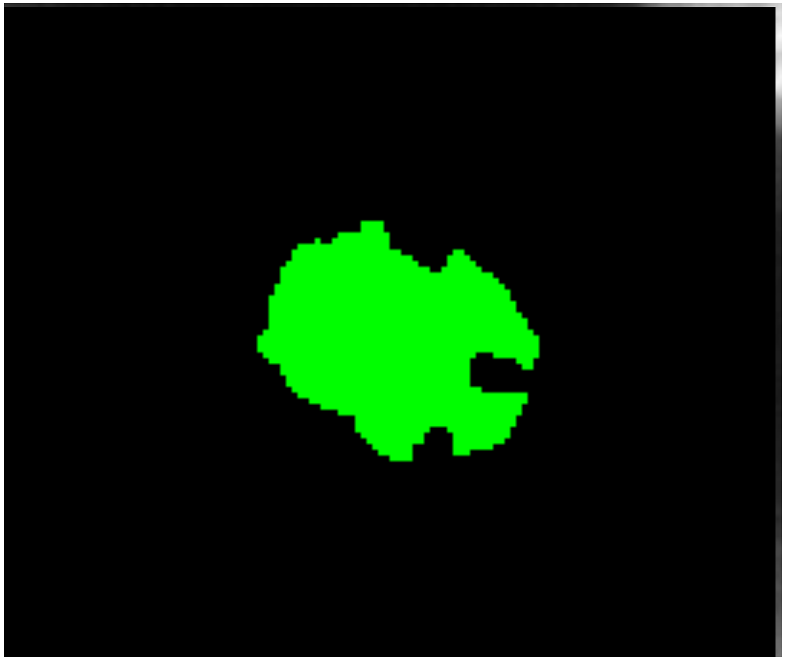


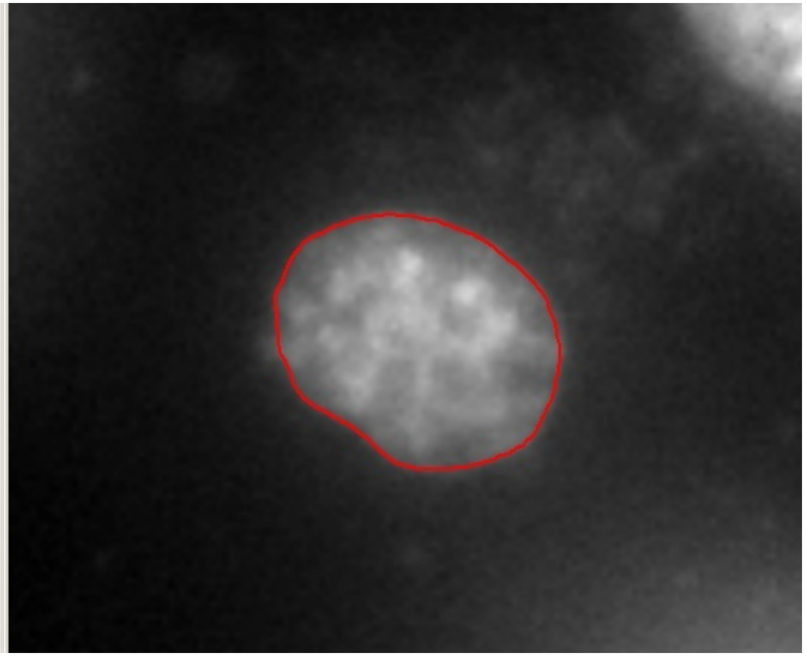
To close:

$$A \bullet B = (A \oplus B) \ominus B$$

Segmentation – basics

- “Some” times more information is needed in order to achieve a good segmentation





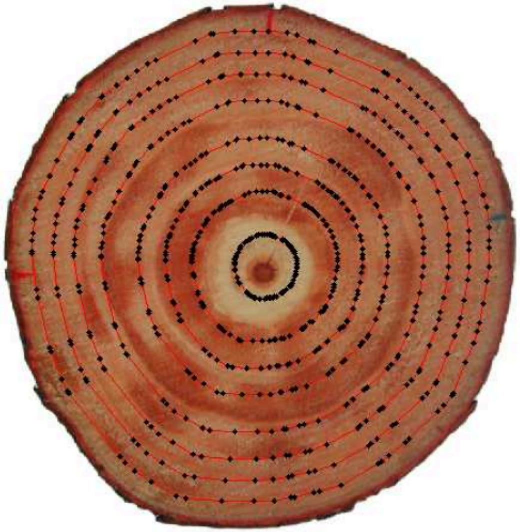
- Template matching
  - "Classic model" Hough transform
  - Applies to circles, line segments and a variety of shapes

Hough P (1959)

If we can detect edges we can approximate a circle (or other shapes)

A test is performed to determine the circles with "best fit"

(c) Sample Tree 8.



(d) Fully automatic recognition with the input figure 5(c).

Cerda M, Hitschfeld N 2007

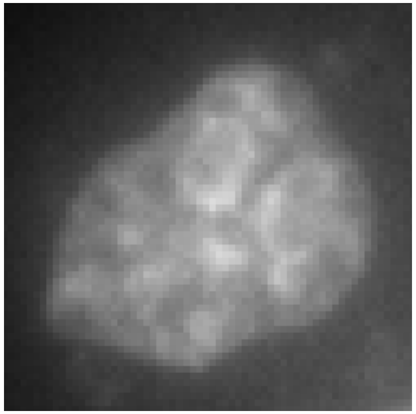
Segmentation - Advanced

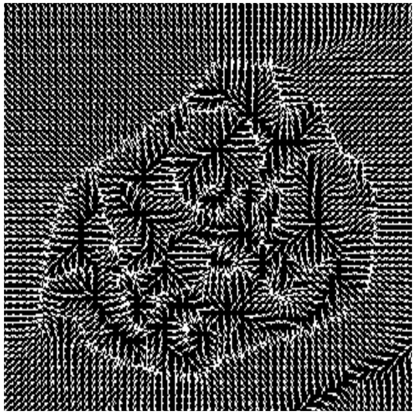
- Variational methods
  - Based on energy minimization, defining integral models
  - Idea: to include desirable features on segmented images (like homogeneus regions, short or smooth ROI boundaries)
  - Optimum solutions found by partial differential equations
  - Examples: Mumford-Shah, Ambrosio-Tortorelli, Chan-Vese (details in the book from Aubert & Kornprobst 2006)

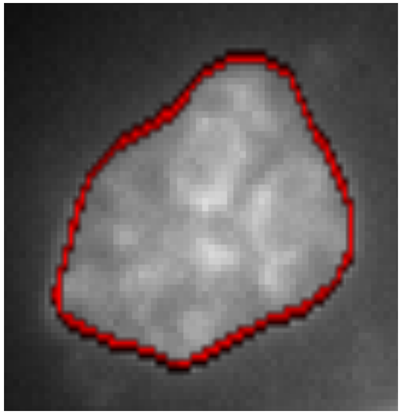
image I | main discontinuties in I | ROI boundaries B | piecewise smooth image J

The Mumford & Shah functional (1989)

- Active contour models
  - Optimization of different properties







input
image
+initial guess

contour C(s)
- elasticity
  (contraction)
- rigidity
  (bending, cornering)

force field
- repulsion
- attraction

output: force balance
minimal energy

First active contours approach:
Kass, Witkin & Terzopoulos (1988)
“Snakes”

Segmentation - Advanced

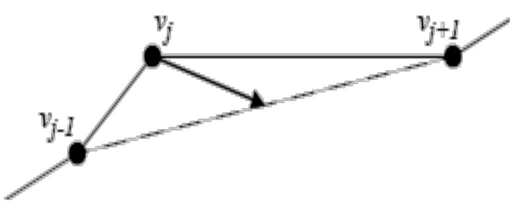
- Snakes: optimization derived from a variational approach
  - Minimization of an integral functional
    “a snake minimizes its energy”

$$E = \int_0^1 \frac{1}{2}\left[\alpha\left|\frac{\partial C(s)}{\partial s}\right|^2 + \beta\left|\frac{\partial^2 C(s)}{\partial s^2}\right|^2\right] + E_{ext}[C(s)]ds$$

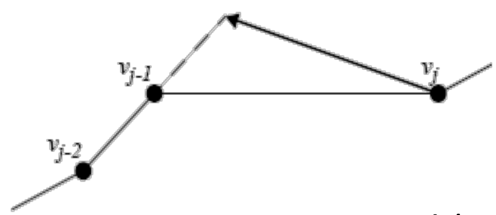
Internal energy, contour dependant

External energy, image dependant

Elasticity term (coefficient α)



Rigidity term (coefficient β)



Kass et al (1988) Snakes: active contour models
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