Image Processing

Shape and topology

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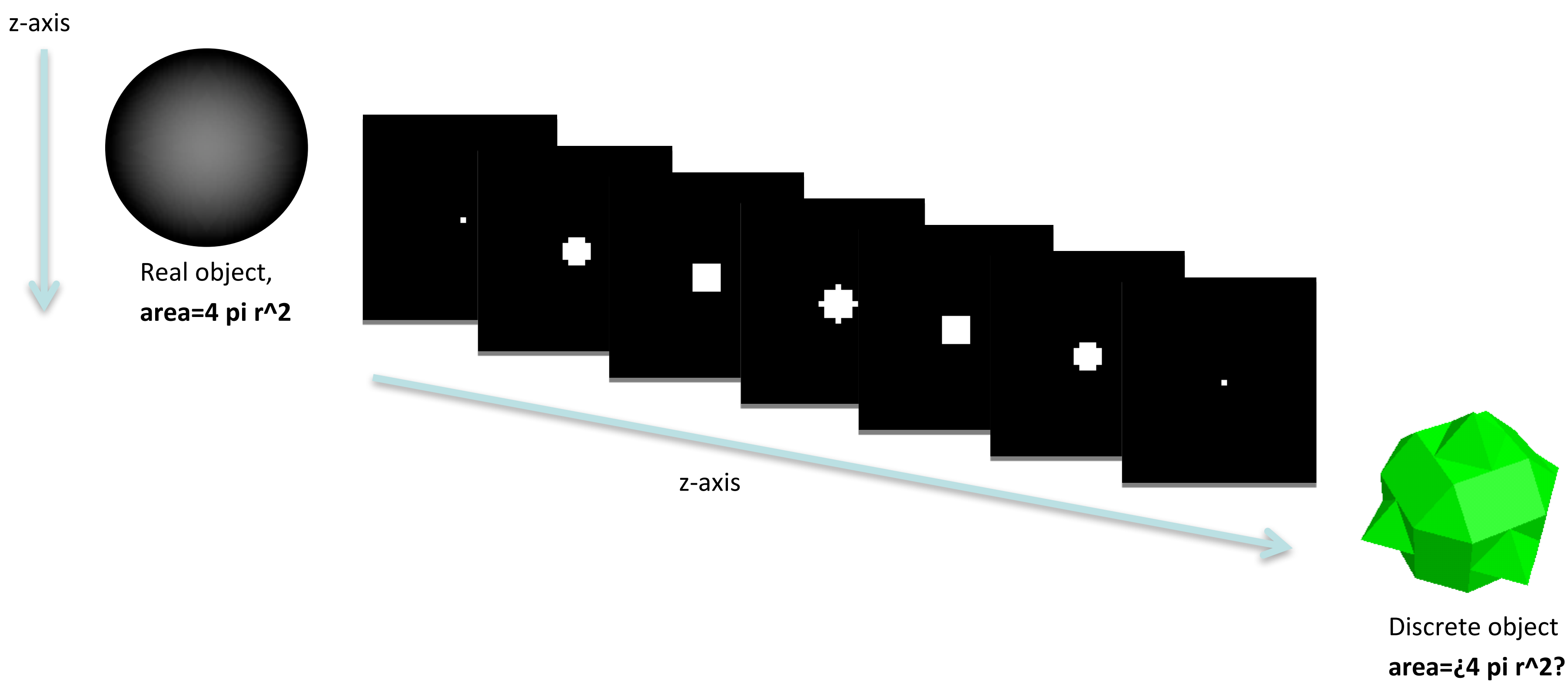
Outline

1. Introduction
    - Images
    - Segmentation

2. Descriptors
    - Shape & Topology*
    - Tracking & more

Motivation

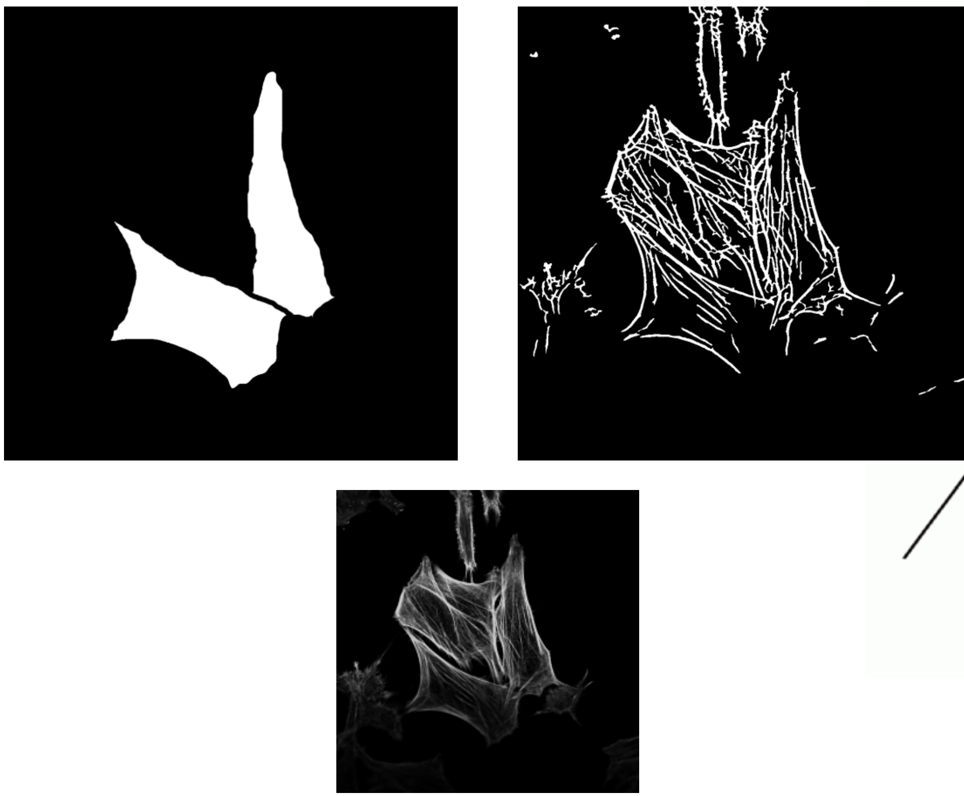
- How to characterize 1 object or compare 2 or more objects?
- How to quantify in a discrete space?
- What error(s) can we expect?



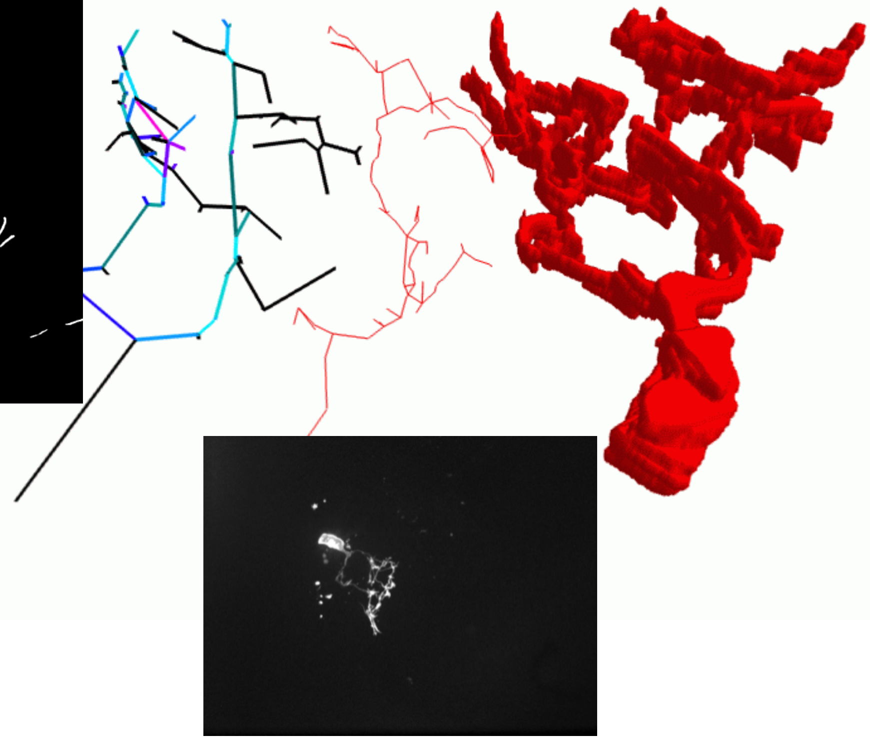
Motivation

Shape / topology / dynamics descriptors

Ex. 1
Shape and structure analysis of fibroblast actin fibers (astrocytes)



Ex. 2
Zebrafish parapineal organ neuron



2D image and segmentations, Lisette Leyton's Lab (BNI)

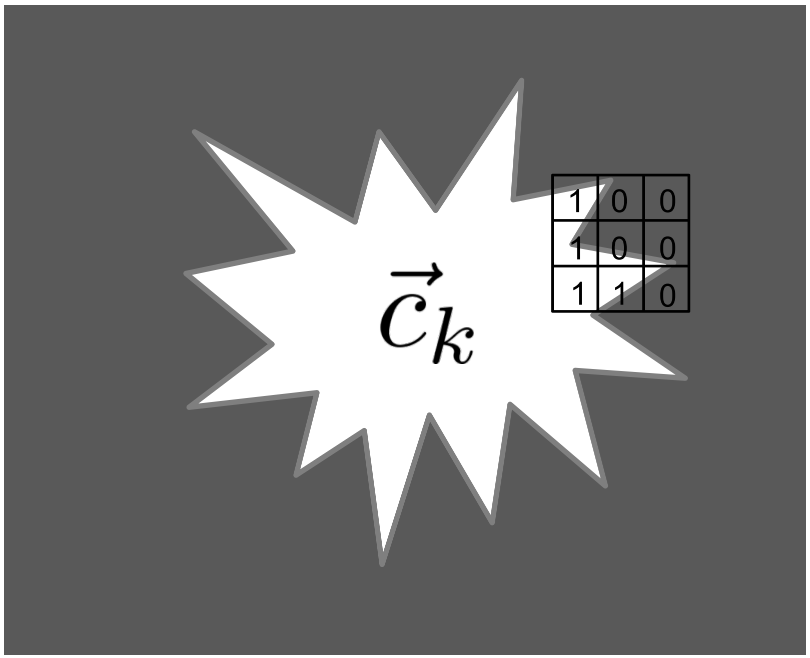
Original image and 3D segmentation , Karina Palma (LEO, BNI).

Shape & topology descriptors…

1. Geometrical descriptors: location, boundary, area, volume
2. Moments of morphology (order 0-2)
3. Topology in computer science (skeletons)

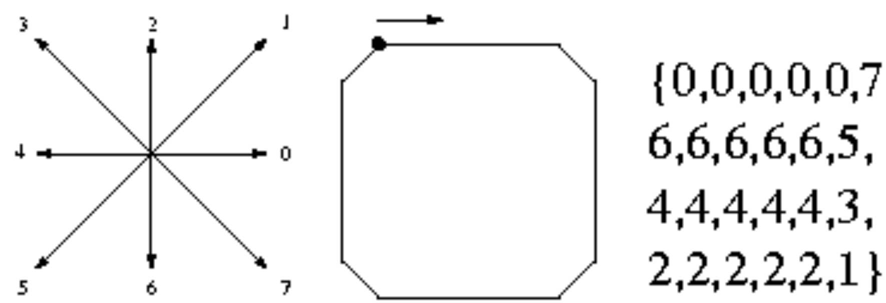
Definitions

A binary ROI from an image stack and its chain code representation.



- Binary image $I(x, y) \in \{0, 1\}$
- ROI coordinates $\vec{c}_k = (x, y)$
- Then $I(\vec{c}_k) = 1$

- Alternative, chain code:

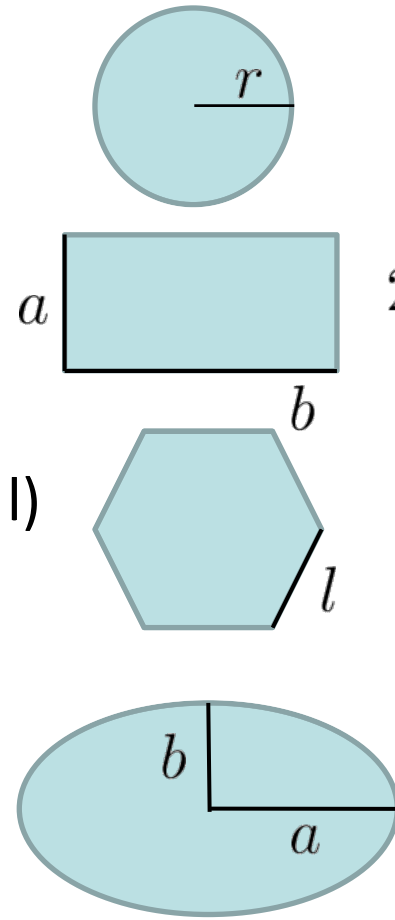


(a) (b) (c)

Geometrical descriptors: boundary

The perimeter is the ROI contour length in 2D. For simple geometries it is well known:

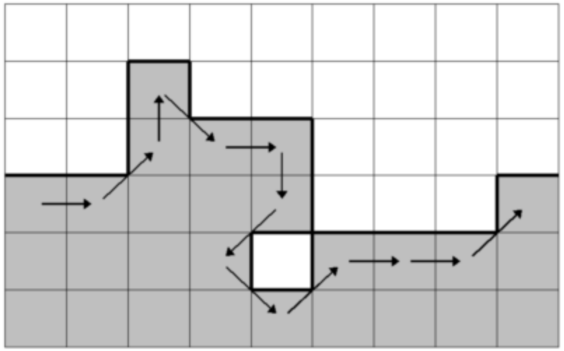
- Circle — $2\pi r$
- Rectangle — $2(a+b)$
- Regular polygon of n faces with length l) — nl
- Ellipse



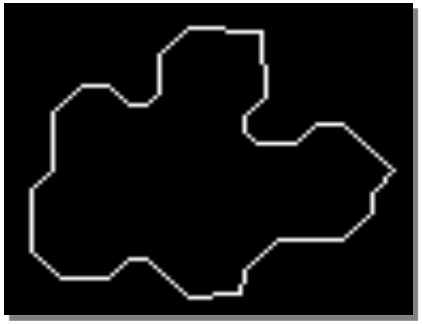
Geometrical descriptors: perimeter

The perimeter can be measured directly from a binary ROI or by using a polygonal representation. How to choose?

- Using chain code each arrow adds 1
- Can this be improved?

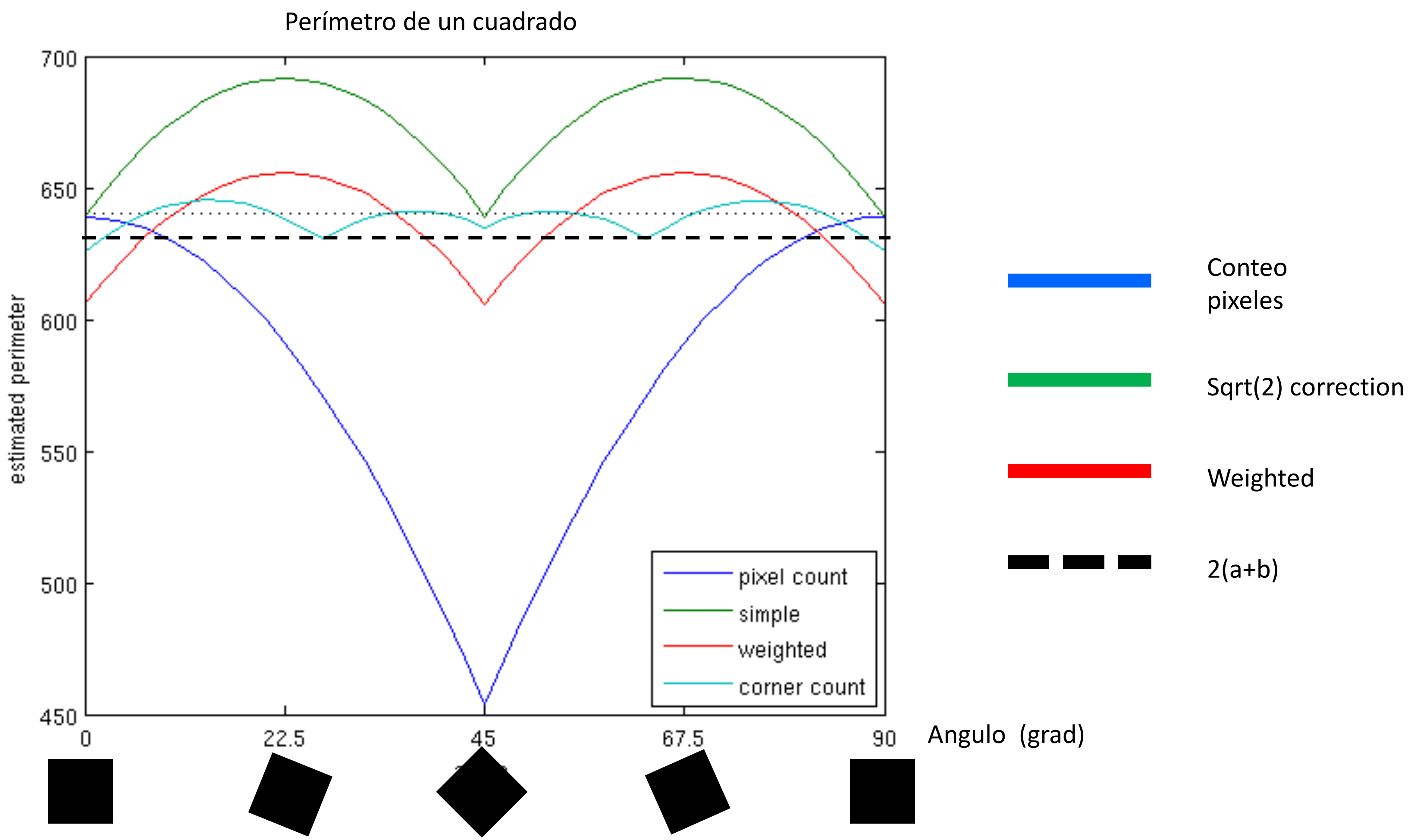


- Using a polygonal representation (e.g. spline curves, active contours), add the elements length



Geometrical descriptors: perimeter

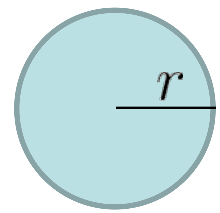
The perimeter can be measured directly from a binary ROI or by using a polygonal representation, how to choose?



Geometrical descriptors: area

The area is the ROI surface measure in 2D.
For simple geometries this is well known:

- Circle

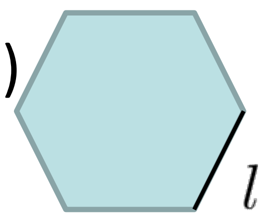


$$\pi r^2$$

- Rectangle

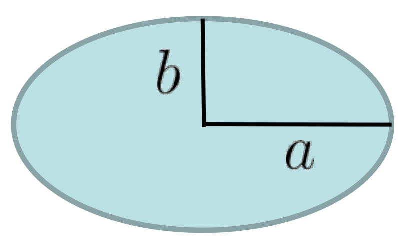
$$ab$$

- Regular polygon of n faces (length l)



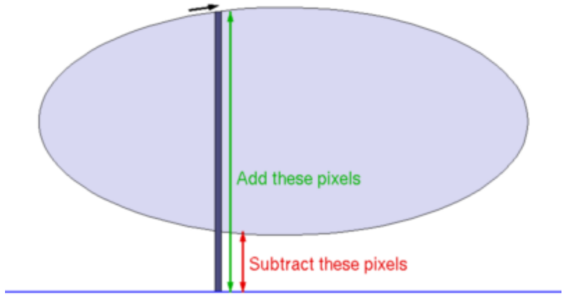
$$n\frac{l^2}{4}\tan\left(\frac{\pi(n-2)}{2n}\right)$$

- Ellipse



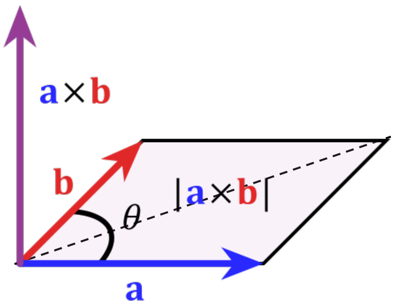
$$\pi ab$$

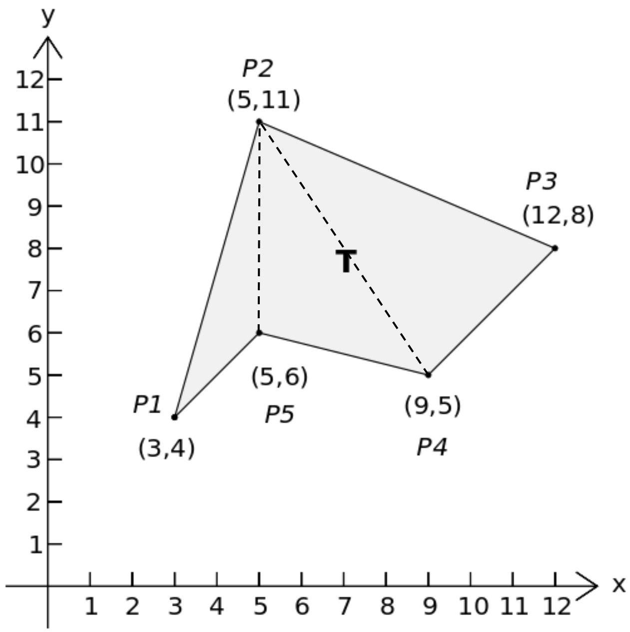
The area can be measured directly from a binary ROI or by using a polygonal representation. How to choose?



- Using a binary ROI
  - Count the number of pixels
  - Using chain code and an extra line

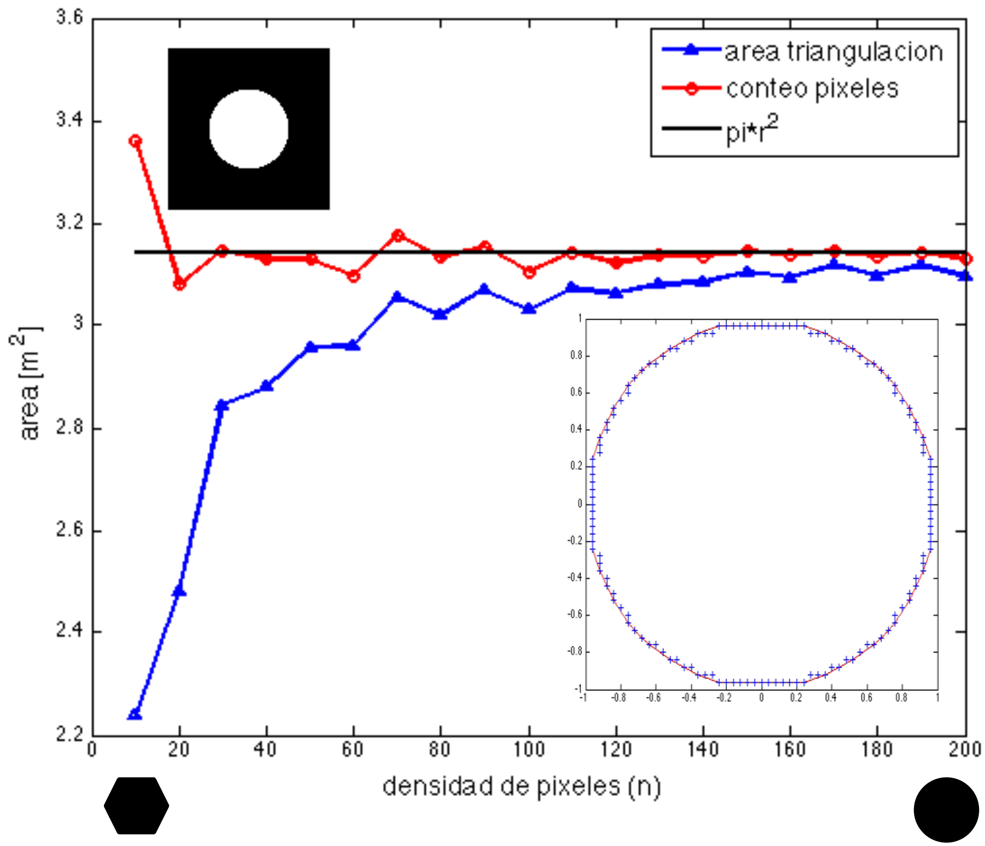
- Using a polygonal representation
  - Using the shoelace algorithm.





Geometrical descriptors: area

The perimeter can be measured directly from a binary ROI or by using a polygonal representation. How to choose?

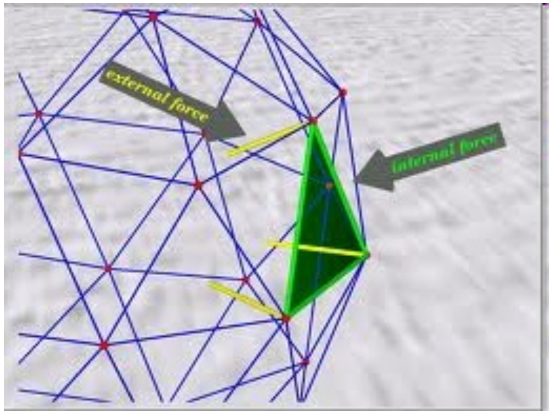


Geometrical descriptors: area

In 3D also the surface area can be measured directly or by using a polygonal representation. How to choose?

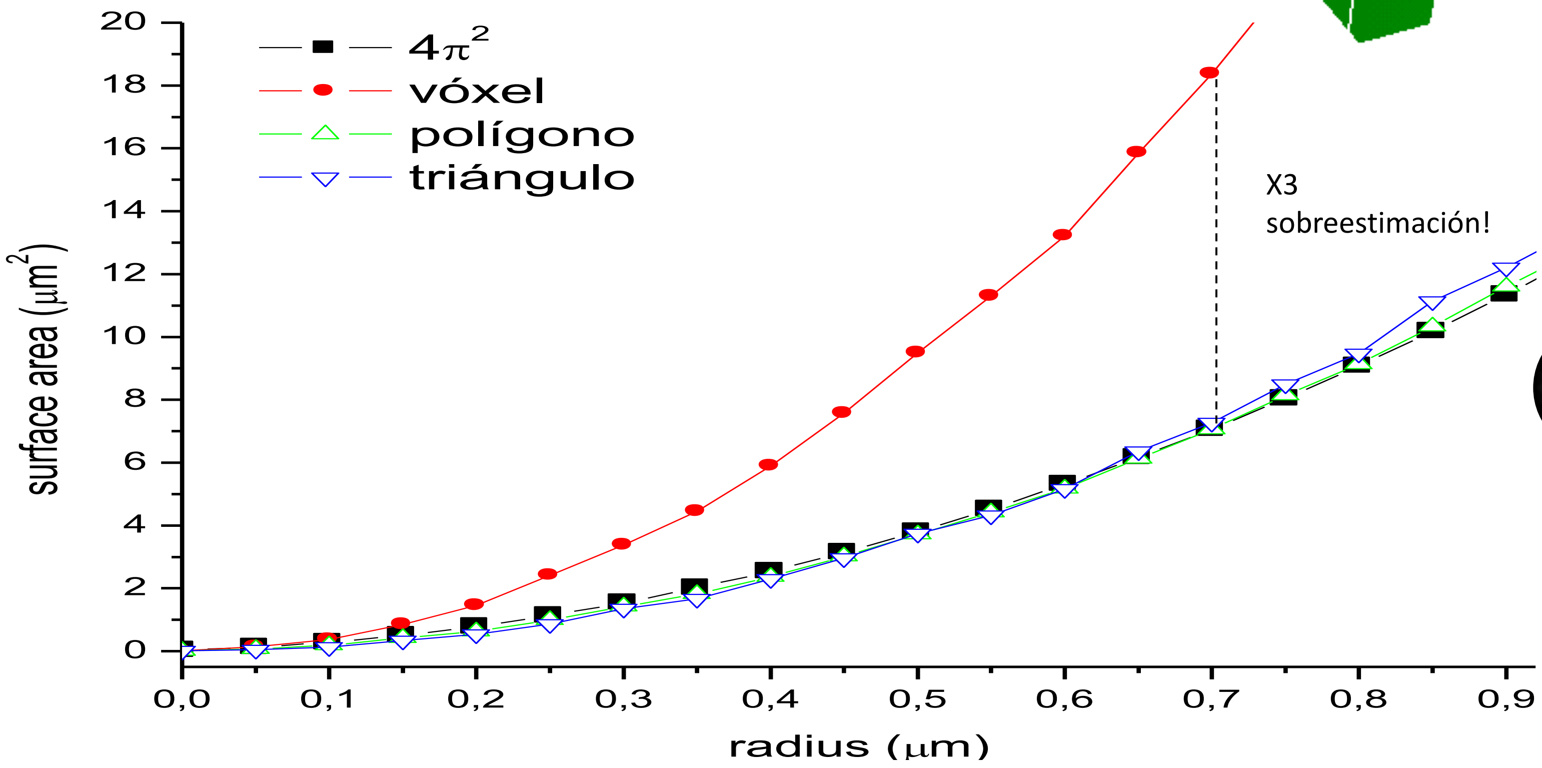
- In a binary ROI, add each voxel of the surface

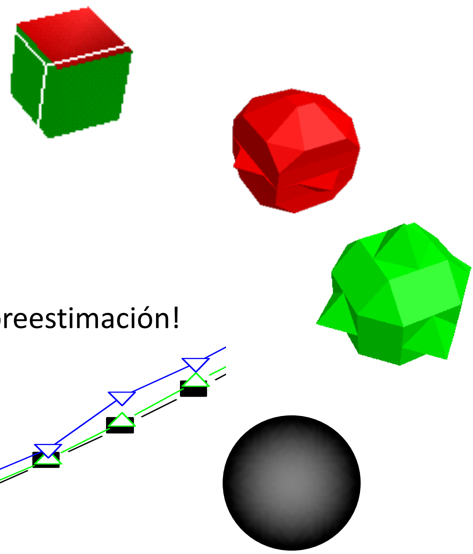
- With surface models, add the area of each element (e.g. triangle)



Geometrical descriptors: area

In 3D also the surface area can be measured directly or by using a polygonal representation. How to choose?





Geometrical descriptors: volume

The volume is the ROI size in 3D. For simple geometries it is well known:

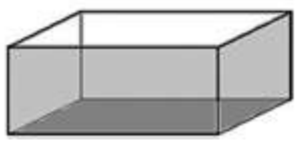
- Sphere

$$\frac{4}{3}\pi r^3$$

- Ellipsoid

$$\frac{4}{3}\pi abc$$

- Parallelepiped

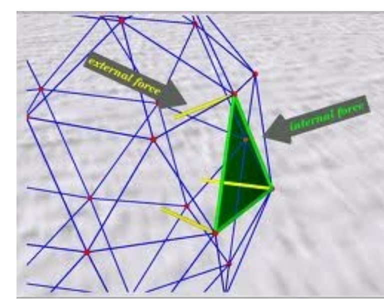


$$abc$$

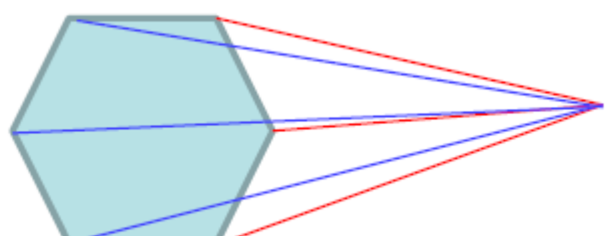
Geometrical descriptors: volume

The volume can be measured directly from a binary ROI or by using a polygonal model.
How to choose?

- In a binary ROI, count the ROI pixels/voxels

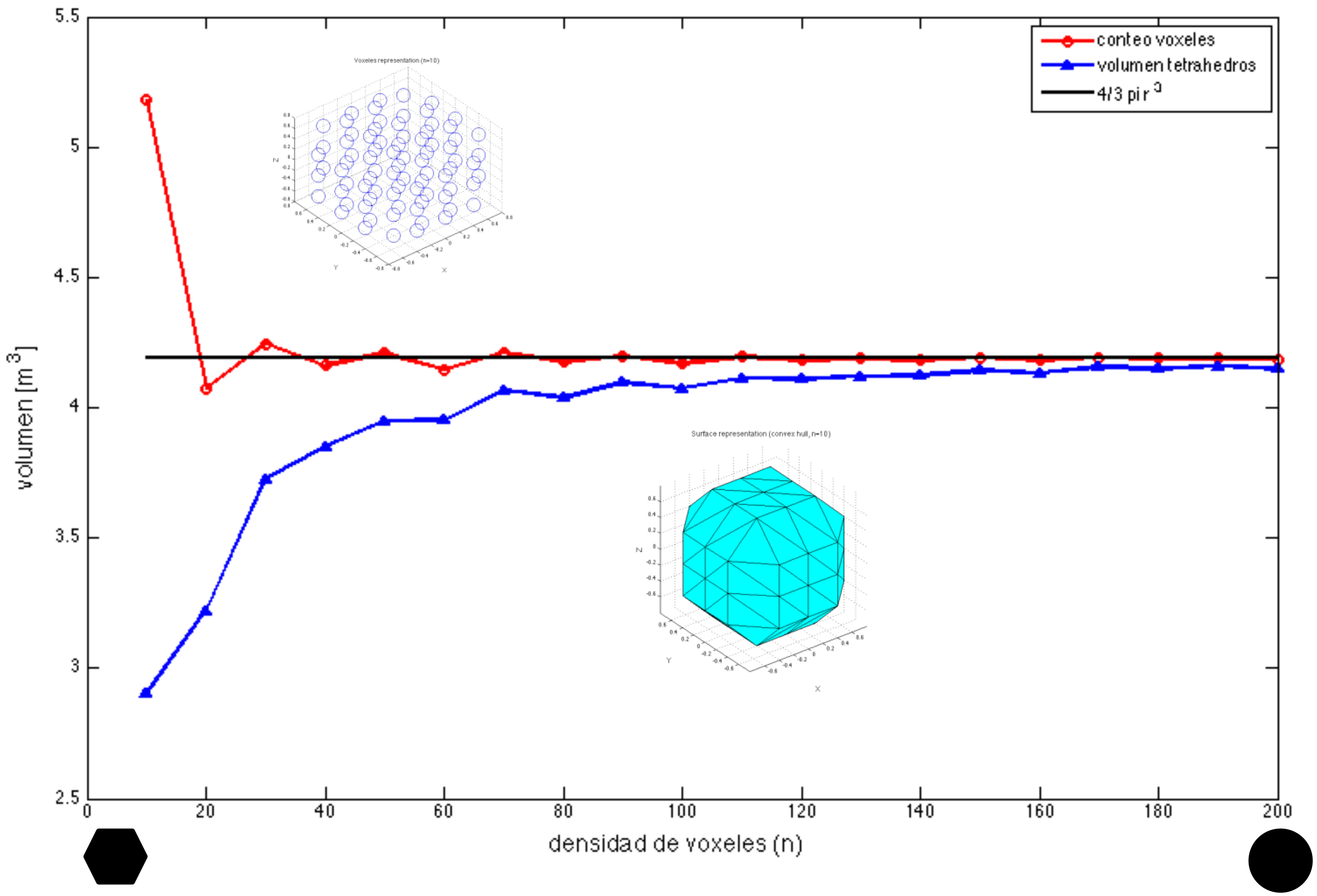


- With triangle surface meshes, add the signed volume of the tetrahedron of each triangle and any point in the space.



Geometrical descriptors: volume

The volume can be measured directly from a binary ROI or by using a polygonal representation. How to choose?



Geometrical descriptors - remarks

Representation and descriptors are tightly related

- For boundary measurements: perimeter (2D), curvature (2D/3D), 3D surface area, geometrical models are more accurate
- For ROI-interior measurements: 2D area, 3D volume, pixel counting /voxel counting are more (or sufficiently) accurate

Shape & topology descriptors…

1. Geometrical descriptors: location, perimeter, area, volume, curvature
2. Moments of morphology (order 0-2)
3. Topology in computer science (skeletons)

Moments of morphology

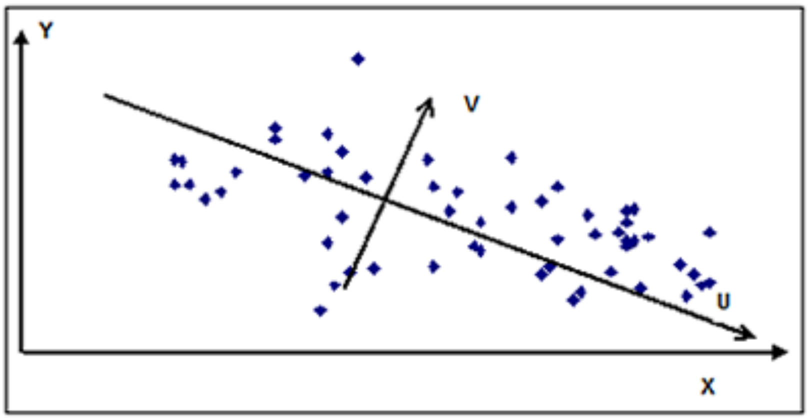
Location, perimeter, area and volume partially describe an object, but not its shape.

Original image (2D),
Prof. Lisette Leyton

How to quantify the amount of roundness of an object?

Moments of morphology

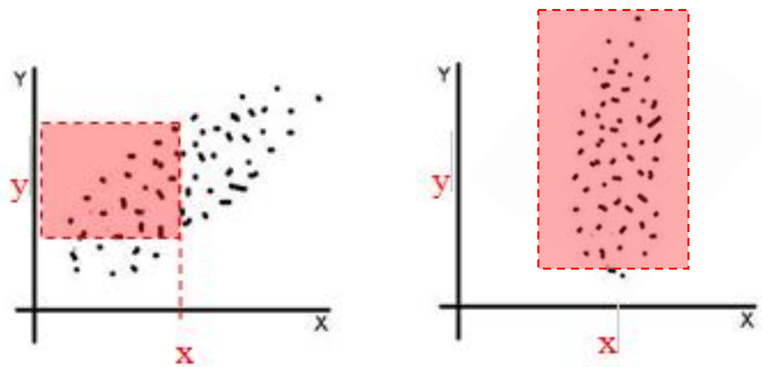
Variance



Axes U and V maximize **the variance in U**

$$\sigma_x^2 = \frac{\sum(x-\overline{x})^2}{N} = \frac{\mu_{2,0}}{\mu_{0,0}}$$

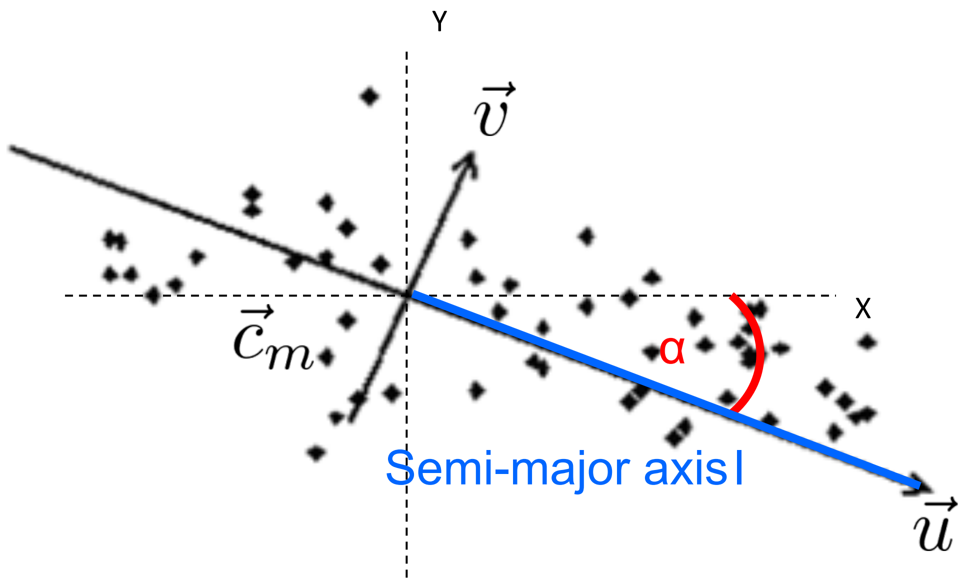
Covariance



Positive covariance   Zero covariance

$$\sigma_{xy}^2 = \frac{\sum(x-\overline{x})(y-\overline{y})}{N} = \frac{\mu_{1,1}}{\mu_{0,0}}$$

We look for two parameters to characterize the binary ROI…



Moments of morphology

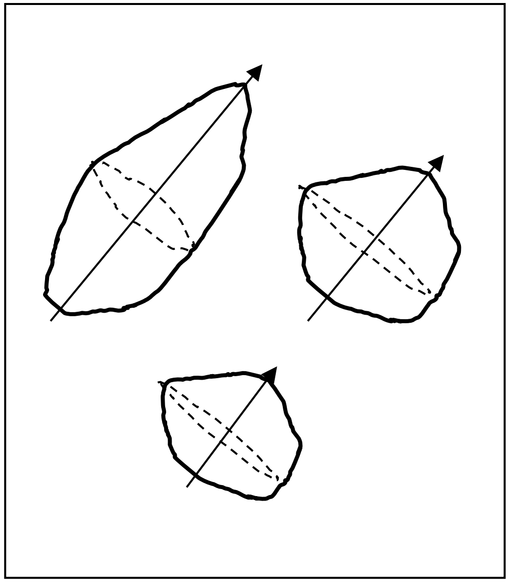
- If the variance/covariance matrix is diagonal for a certain rotation α,

- Semi-major axis length *l* is a function of second-order moments

$$l^2 = \lambda = \frac{1}{2}\left(\sigma_{xx}^2 + \sigma_{yy}^2 + \sqrt{(\sigma_{xx}^2 - \sigma_{yy}^2)^2 + 4(\sigma_{xy}^2)^2}\right))$$

Moments of morphology

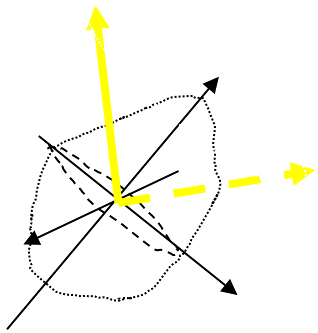
Second order moments of morphology describe a ROI main axis (principal components)



- The **major axis** corresponds to the direction with the biggest variance.
- The **secondary axis (minor in 2D)** is orthogonal to the major axis, giving in 3D the direction of the second biggest variance.
- The **third axis** (3D) is orthogonal to the major and secondary axes.

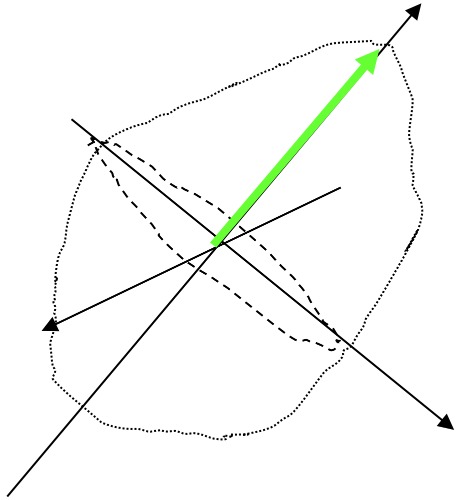
Moments of morphology

Principal axes are useful object descriptors because…



- they directly define the object *length*, *height*, and *width*

- using principal axes, similarities between objects can be found

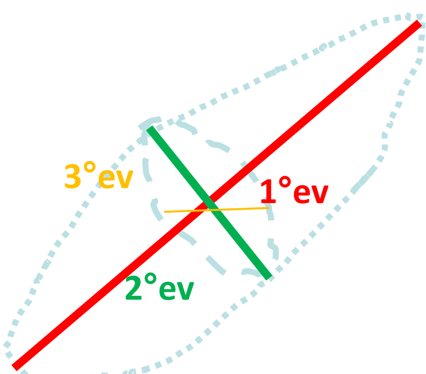


By combining the principal axes, we can define *Elongation*, *Relative Elongation*, and *Flatness*.

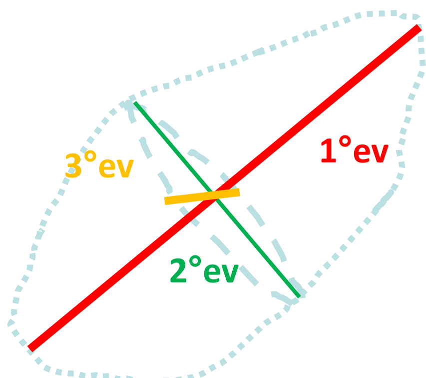
$$Elong = 1 - \frac{2°ev}{1°ev}$$

$$R.Elong = 1 - \frac{3°ev}{1°ev}$$

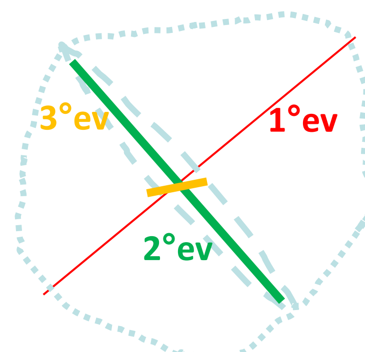
$$Flatness = 1 - \frac{3°ev}{2°ev}$$



1°ev >> 2°ev
Elong. ~ 1



1°ev >> 3°ev
R. Elong. ~ 1



2°ev >> 3°ev
Flatn. ~ 1

Moments of morphology - remarks

- The center of mass allows to describe each object position
- The variance allows to computer principal axes (eigenvectors) and to quantify dispersion for each principal axis direction (eigenvalues)
- Higher order moments describe more detailed information like asymmetry or kurtosis
- Composed parameters between eigenvalues deliver morphological parameters like elongation or flatness

Shape & topology descriptors…

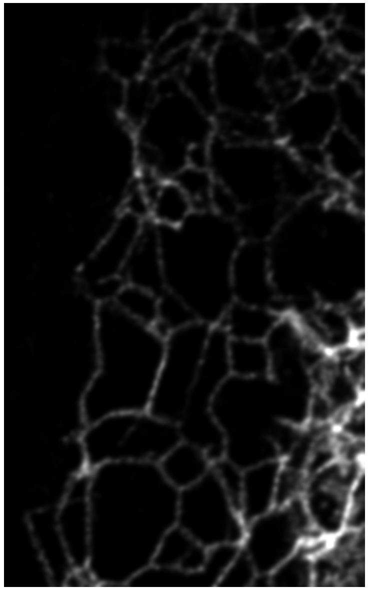
1. Geometrical descriptors: location, boundary, area, volume, curvature
2. Moments of morphology (order 0-2)
3. Topology in computational geometry (skeletons)

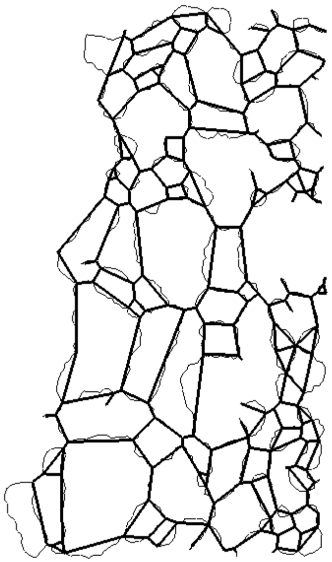
Topology

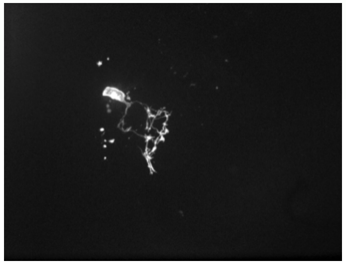
How to characterize complex biological structures?

Patterns, constraints? (branching direction, number of branches)

Ex. neurons, endoplasmic reticulum.







Original image and segmentation from Omar Ramírez and Jorge Toledo (SCIAN-Lab, BNI).

Original image and segmentation (3D) from Karina Palma (LEO-SCIAN, BNI)

Skeleton definition

- A reduced-dimension object representation (e.g. only lines)
- Approximately equidistant to the object contour
- Aim: to retain and ease quantification of geometrical and topological properties like connectivity, length, width and tunnels

Laboratory for SCIAN Scientific Image Analysis

Skeleton definition

Biomedical Neuroscience Institute

Skeleton main problem: several definitions

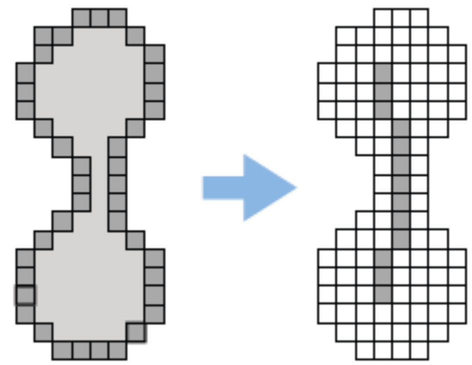
More accepted definitions:

- Medial axes, medial surfaces
- Center of maximal discs (2D) / spheres (3D)

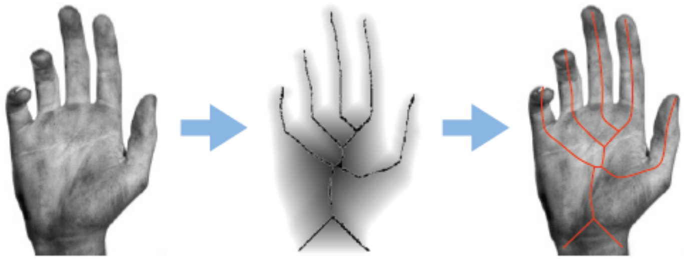
- Many “correct” skeletons can be obtained from the same ROI
- Moreover, each definition is more suitable for a given problem, and many skeletonization algorithms exist

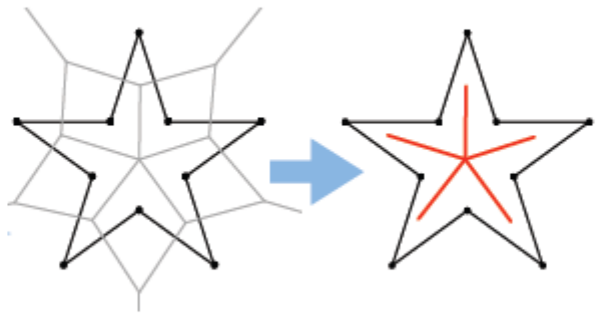
Skeletonization algorithms

Skeletonization algorithms can be mainly classified in 3 groups:



- Morphological (thinning)
- Graph based
- Geometrical

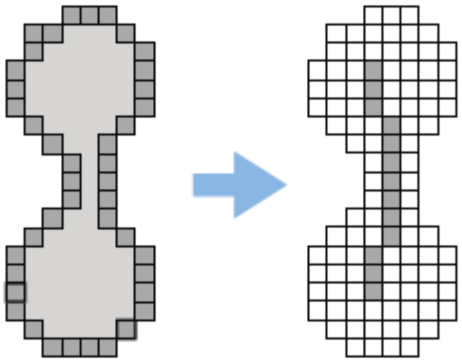




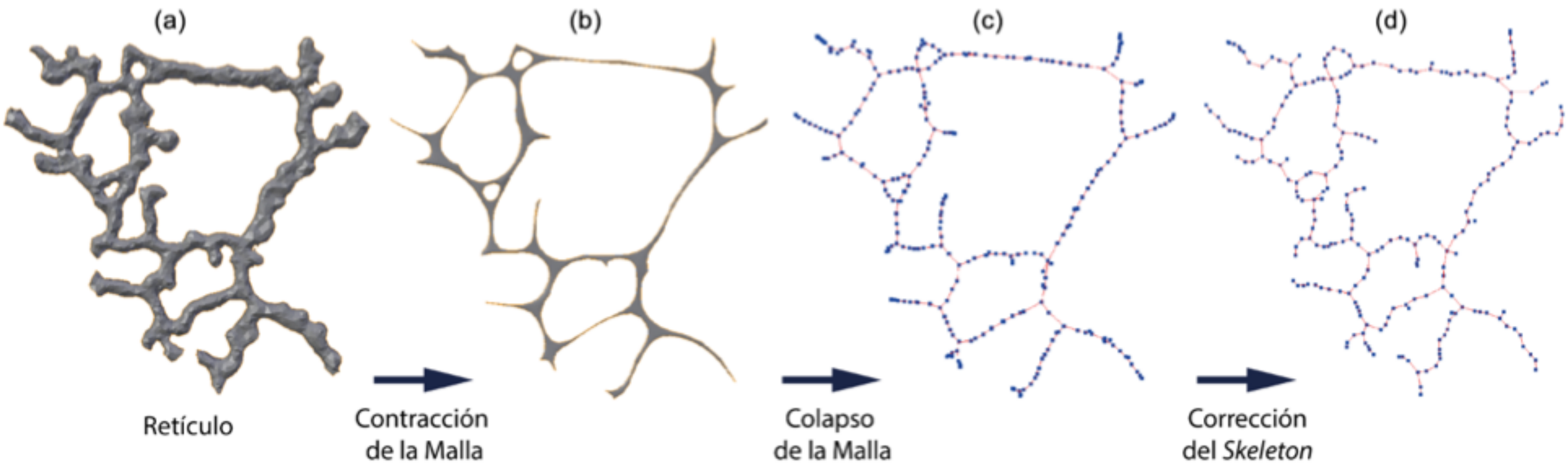
Morphological skeletons (thinning)

Idea: to remove pixels/voxels from the contour until the ROI width is 1.

- Only work for pixel/voxel representations (not polygons).
- Result is not centered
- Noise sensitive
- Fast
- Easy to implement and find (ImageJ)



Geometry contraction skeletons



Idea: contract a 3D object model until a 1D object is reached:

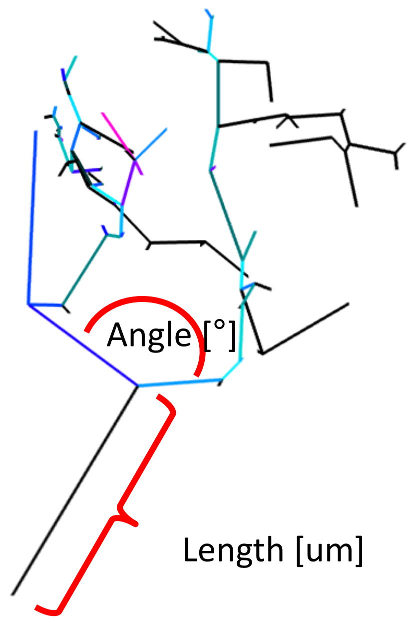
- Centered skeleton
- Robust to noise
- Much more computationally intensive
- Implemented and improved in our lab (SCIAN-Lab)

Skeleton extraction by mesh contraction
Au et al. 2008 ACM SIGGRAPH
Alcayaga 2012, Rojas 2014 DCC, SCIAN-Lab
[Lavado 2016]

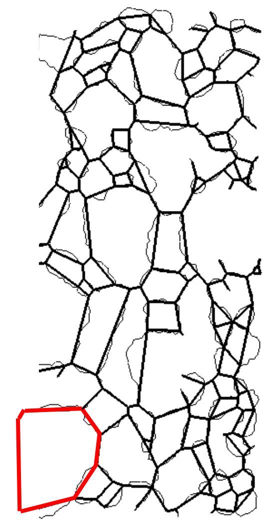
Skeleton descriptors

What information can be obtained from a skeleton (a graph in the broad sense)?

- Node number
- Arc (edge or segment) count
- Tunnels number
- Arcs length
- Bifurcation degree



Palma et al, 2012



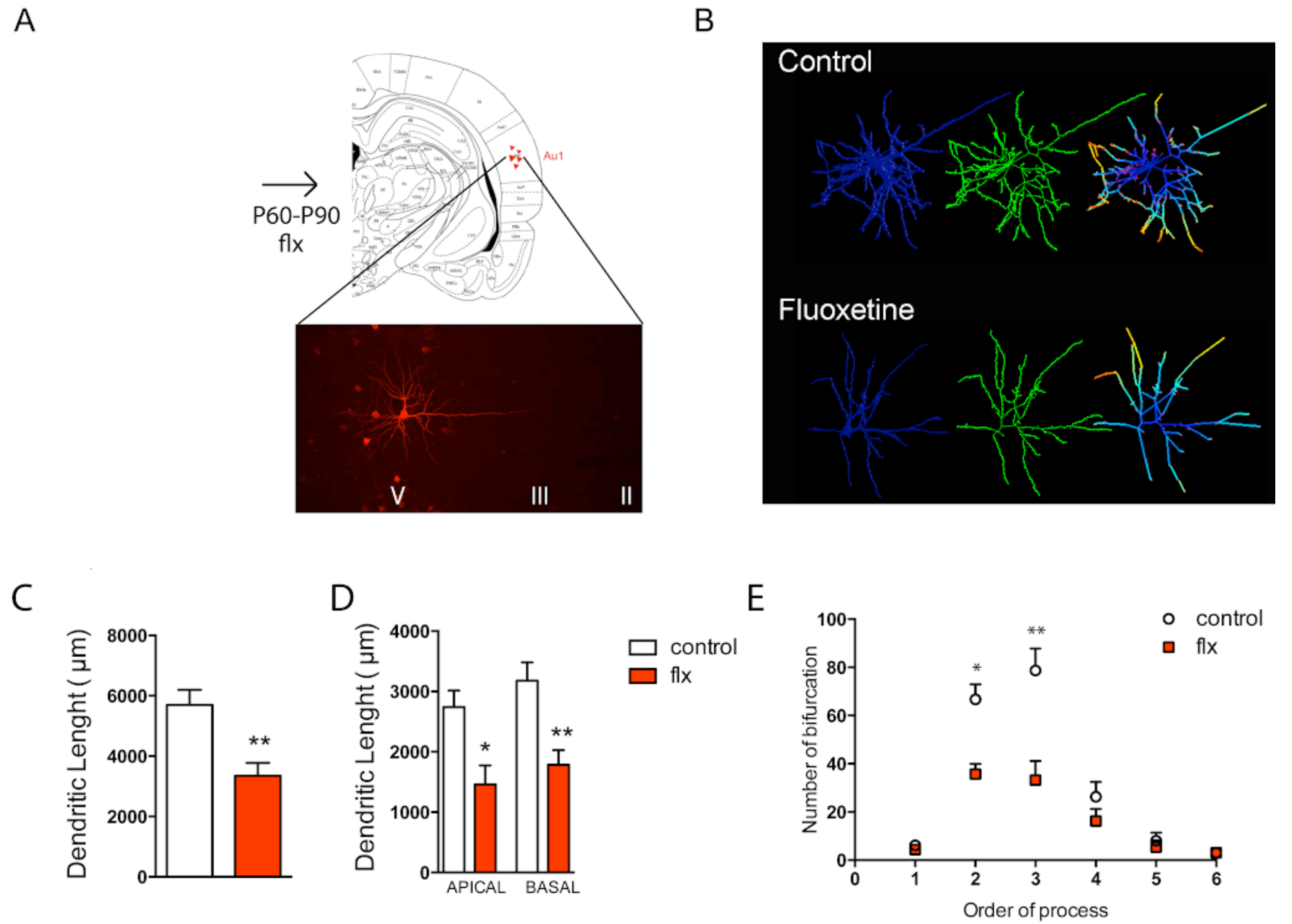
Ramirez et al., 2012

Advanced topology descriptors

Other descriptors, inspired from graph theory and complexity…

- Sholl complexity (Sholl analysis)
- Shortest path
- Entropy
- Kolmogorov complexity

Skeletons - applications



Ampuero et al 2016